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Evidential Reasoning approach for multiple-criteria decision making: A simulation-based formulation

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ABSTRACT

Multiple-criteria decision making (MCDM) permeates in almost every industrial and management setting. The Evidential Reasoning (ER) approach, pioneered and developed by Yang, Xu and their colleagues since the early 1990's and currently with applications in a wide ranging set of domains, is among the premier methods for MCDM. While it is hard to dispute the versatility of the ER approach, a key disadvantage in the existing ER framework is that its formulation involves complex formulas with logically non-trivial proofs. This complexity forces the non-specialists to use ER as a black-box technique, and presents definite impediment for the specialists to further develop ER. A contribution of the present article is that through a conceptually simple recasting of ER into a simulation-based framework (termed *SB-ER*), we show that the complexity seen in the existing ER framework can be radically reduced – it now becomes logically straightforward to comprehend the inner working of ER. Further, we show that the capability of the existing ER approach can be readily extended via this simulation-based framework. Thus, owing its intellectual debt to and building upon the firm foundation of ER, SB-ER paves a promising shortcut for fine-tuning and further developing ER. Finally, we demonstrate the utility of SB-ER using a small industrial dataset. To facilitate further development, a set of Matlab source codes, which complements currently available ER-based software, is available from the author upon request.

1. Introduction

Decision making in almost all industrial and institutional settings involves evaluating alternatives with respect to multiple conflicting criteria and then choosing the "best" alternative based on the evaluation results. For example, in appraising proposals to improve the overall effectiveness of emergency room services in a public hospital, service level, cost and employees' workloads are just three of the many conflicting factors that need to be taken into account. Researchers working in the domain of multiple-criteria decision making (MCDM) have been developing a rich and varied set of methods to aid professionals in arriving at sound decisions for such types of problems. The Evidential Reasoning (ER) approach, the focus of the present article, is one of the premier methods for tackling MCDM problems. Since first being introduced in Zhang, Yang, and Xu (1989), Yang and Singh (1994) and Yang and Sen (1994), ER has been extensively developed (e.g. Guo, Yang, Chin, and Wang (2007), Guo, Yang, Chin, Wang, and Liu (2009), Xu, McCarthy, and Yang (2006), Xu, Yang, and Wang (2006)) and demonstrated in various application domains (e.g. Chin, Yang, Guo, and Lam (2009), Graham and Hardaker (1999), Hilhorst, Ribbers, Heck, and Smits (2008), Kabak and Ruan (2011), Liu, Ruan, Wang, and Martinez (2009), Martinez, Liu, Ruan, and Yang (2007), Ren, Yusuf, and Burns (2009), Sonmez, Graham, Yang, and Holt (2002), Tanadtang, Park, and Hanaoka (2005), Xu et al. (2006), Yang, Xu, Xie, and Maddulapalli (2011), Yao and Zheng (2010)). As argued in Xu (2012), traditional MCDM approaches such as AHP (Saaty, 1988) do not have explicit mechanism to represent uncertainties such as ignorance. In contrast, ER is firmly grounded on the Dempster–Shafer evidence theory (Shafer, 1976) and possesses the added notions of belief structure and belief decision matrix (Xu & Yang, 2003; Yang & Xu, 2002). Therefore, it is intrinsically capable to represent various kinds of uncertainties and ignorance in a natural and integrated manner, even if a given probabilistic model is incomplete.

While it is hard to dispute the versatility of the ER approach as evidenced by the above-mentioned utilization of ER in a broad range of application domains, one disadvantage with the approach remains: The existing formulation of the core ER framework (Yang & Sen, 1994; Yang & Singh, 1994; Yang & Xu, 2002) and its various extensions (e.g. Guo et al. (2009), Wang et al. (2006a, 2006b), Xu

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et al. (2006), Yang, Wang, Xu, and Chin (2006)) all involve complex formulas together with logically non-trivial proofs (e.g. see the appendices of Guo et al. (2009) and Xu et al. (2006)). As such, a non-specialist who desires to use the ER approach to solve decision problems in his/her application domains will have to rely on ER as a black-box technique. If ER happens to give an unexpected outcome, the non-specialist will have no easy means to trace the source of the unexpectedness or to fine-tune the approach for his/her specific situation. Thus, it is conceivable that the more circumspect professionals may even be hesitant to use ER altogether due to the lack of understanding of its inner working and hence unsure about the suitability of ER to their problems. In order to make the ER approach attractive to a wider range of users, it is imperative that the formulation of the ER framework be made more transparent and intuitive. Moreover, even for the experts, an alternative formulation offers an additional vantage point to appreciate the theoretical landscape of ER, and this can speed up further theoretical development of ER and facilitate its combinations with other decision support methods.

Thus, a goal of this article is to provide a reformulation of the ER technique that is easy to understand. Our approach is to recast ER into a simulation framework (herein termed *simulation-based ER*, SB-ER for short), by employing the random-switch metaphor of Pearl (1988), p. 416 to model ER. As a consequence, MCDM outcomes that closely approximate those generated from the formal ER approach (as exemplified in the references mentioned in the preceding paragraph, herein termed *formal ER*) can be generated via computer simulation. Moreover, formulas previously derived using formal ER can be shown to emerge out naturally by thinking in terms of simulation. Last but not least, we also give several examples of how to further develop ER via simulation-based thinking. In short, the complexity encountered in the existing ER framework can be significantly reduced: It now becomes logically straightforward to comprehend the inner working of ER, and to fine-tune and further develop ER.

The rest of this article is organized as follows: in Section 2, for completeness, the notations and the bare essentials of the formal ER approach will be summarized. We devote Section 3 to introducing the simulation-based ER framework. Also, using SB-ER, formulas obtained in previous analytical works will be re-derived and examples of possible further extensions of the ER technique will be discussed. In Section 4, we illustrate and validate SB-ER by analyzing an industrial engineering MCDM dataset. Finally, a brief summary will be described in Section 5.

2. Background

2.1. The basics of the formal ER technique

For concreteness, consider a simplified version of an MCDM problem from Yang and Xu (2002) that the formal ER technique is designed to solve: we would like to evaluate the handling quality of two motorcycle models (i.e. two entities), say Kawasaki and Honda respectively. Suppose that the *handling quality* attribute comprises of three criteria: *steering*, *maneuverability* and *top speed stability*. The non-negative weights w_1, w_2 and w_3, such that $w_1 + w_2 + w_3 = 1$, have been assigned to the three criteria respectively to represent the relative importance of these criteria in determining the overall handling quality. After gathering opinions from experts, suppose that the grades as described in Table 1 are given to Kawasaki and Honda. The individual grades H_1, H_2 and H_3 stand for *Below Average*, *Average*, and *Good* correspondingly. Now, take the two entries from the top row of Table 1 as examples – "H_1 (0.5) and H_2 (0.5)" indicates that the steering of Kawasaki is considered "below average" to a belief degree of 0.5 and "average" to a belief degree of 0.5, whereas "H_2 (0.5) and H_3 (0.3)" means that the steering of Honda is considered "average" to a belief degree of 0.5 and "good" to a belief degree of 0.3. An assessment is said to be complete if the total belief degree equals 1 (e.g. the assessment about the steering of Kawasaki, being 0.5 + 0.5 = 1) and incomplete if the total is less than 1 (e.g. that of Honda). In the case of Honda, 1 – (0.5 + 0.3) = 0.2 represents the belief degree not assigned to any individual grade due to ignorance.

Table 1
The ratings for the handling quality of two motorcycle types by the experts. The handling quality is assessed by three criteria: *steering*, *maneuverability* and *top speed stability*. The grades H_1, H_2, H_3 stand for *below average*, *average*, and *good* respectively. The value inside the parenthesis after a grade indicates the corresponding belief degree. For example, the *steering* of Honda is rated *average* with a belief degree of 0.5 and *good* with a belief degree of 0.3.

Criteria	Weights	Motorcycle types	
		Kawasaki	Honda
Steering	w_1	$H_1(0.5)$, $H_2(0.5)$	$H_2(0.5)$, $H_3(0.3)$
Maneuverability	w_2	$H_2(1.0)$	$H_1(0.5)$, $H_2(0.5)$
Top Speed Stability	w_3	$H_3(0.8)$	$H_3(1.0)$

In general, consider an MCDM problem, in which the attribute of interest is composed of L criteria (with non-negative weights $w_1, \ldots, w_L$ such that $\sum_{i=1}^{L} w_i = 1$ given to these criteria), and a set of individual grades $H = \{H_n : n = 1, \ldots, N\}$ is available to rate a given entity with respect to a criterion. Borrowing the notation used in Yang and Xu (2002), $\beta_{n,i}$ represents the belief degree assigned to the grade H_n in assessing such an entity with respect to the ith criterion. On the other hand, $\beta_{H,i} \equiv 1 - \sum_{n=1}^{N} \beta_{n,i}$ denotes the degree of ignorance, and is assumed to be assigned to the complete set of grades H. The following set of formulas can then be employed recursively to combine the assessment results of the L criteria (see Xu (2012) for a synopsis and Yang and Xu (2002) for a complete derivation):

Initialization :

$$I_{n,1} \equiv m_{n,1} (\text{for } n = 1, \ldots, N) \tag{1a}$$

$$I_{H,1} \equiv m_{H,1} \tag{1b}$$

$$\widetilde{I}_{H,1} \equiv \widetilde{m}_{H,1} \tag{1c}$$

$$\bar{I}_{H,1} \equiv \bar{m}_{H,1} \tag{1d}$$

Recursion: (from $i = 1$ until $i = L - 1$)

$$K_{i+1} \equiv \sum_{q=1}^{N} \sum_{\substack{p=1 \\ p \neq q}}^{N} I_{q,i} m_{p,i+1} \tag{2a}$$

$$I_{n,i+1} = \frac{1}{1-K_{i+1}} \{ I_{n,i} \cdot m_{n,i+1} + I_{H,i} \cdot m_{n,i+1} + I_{n,i} \cdot m_{H,i+1} \} \ (\text{for } n = 1, \ldots, N) \tag{2b}$$

$$\widetilde{I}_{H,i+1} = \frac{1}{1-K_{i+1}} \left\{ \widetilde{I}_{H,i} \cdot \widetilde{m}_{H,i+1} + \bar{I}_{H,i} \cdot \widetilde{m}_{H,i+1} + \widetilde{I}_{H,i} \cdot \bar{m}_{H,i+1} \right\} \tag{2c}$$

$$\bar{I}_{H,i+1} = \frac{1}{1-K_{i+1}} \{ \bar{I}_{H,i} \cdot \bar{m}_{H,i+1} \} \tag{2d}$$

$$I_{H,i+1} = \bar{I}_{H,i+1} + \widetilde{I}_{H,i+1} \tag{2e}$$

where

$$m_{n,i} \equiv w_i \cdot \beta_{n,i} \tag{3a}$$

$$m_{H,i} = 1 - w_i \sum_{n=1}^{N} \beta_{n,i} \tag{3b}$$

$$\bar{m}_{H,i} = 1 - w_i \tag{3c}$$

and $$\widetilde{m}_{H,i} = w_i \left\{ 1 - \sum_{n=1}^{N} \beta_{n,i} \right\} \tag{3d}$$