Expert Systems with Applications 38 (2011) 11560–11572

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A unified model between the weighted average and the induced OWA operator

José M. Merigó *

Department of Business Administration, University of Barcelona, Av. Diagonal 690, 08034 Barcelona, Spain

ARTICLE INFO

Keywords:
Weighted average
OWA operator
Aggregation operators
Multi-person decision-making

ABSTRACT

We present a new model that uses the weighted average (WA) and the induced ordered weighted averaging (IOWA) operator in the same formulation. We call it the induced ordered weighted averaging–weighted average (IOWAWA) operator. We study some of its main properties and we see that it has a lot of particular cases such as the WA and the OWA operator. The main advantage of the IOWAWA operator is that it unifies the IOWA operator with the WA in the same formulation considering the degree of importance that each concept has in the aggregation. We analyze the applicability of this new approach and we see that it is very broad because it can be applied in a wide range of fields such as statistics, economics, decision theory and engineering. Theoretically, we could state that all the previous models and applications based on the WA and the IOWA can be revised and improved with this new approach because they will be included in this framework as a particular case. We focus on an application in a multi-person decision-making in political management.

1. Introduction

In the literature, we find a wide range of methods for aggregating the information (Beliakov, Pradera, & Calvo, 2007; Merigó, 2008; Torra & Narukawa, 2007; Yager & Kacprzyk, 1997). The weighted average (WA) is one of the most common aggregation operators found in the literature. It can be used in a wide range of different problems including statistics, economics and engineering. Another interesting aggregation operator that has not been used so much in the literature is the ordered weighted averaging (OWA) operator (Yager, 1988). The OWA operator provides a parameterized family of aggregation operators that range from the maximum to the minimum. Since its introduction, the OWA operator has been studied in a wide range of applications (Ahn, 2009; Alonso, Cabrerizo, Chiclana, Herrera, & Herrera-Viedma, 2009; Chang & Wen, 2010; Cheng, Wang, & Wu, 2009; Kacprzyk & Zadrozny, 2009; Liu, Cheng, Chen, & Chen, 2010; Merigó & Gil-Lafuente, 2008, 2010; Xu, 2009; Yager, 1993, 1996a, 2006, 2007, 2009a, 2009b; Yager & Kacprzyk, 1997; Zhao, Xu, Ni, & Liu, 2010).

A very practical extension of the OWA is the induced OWA (IOWA) operator (Yager & Filev, 1999). It is an extension of the OWA operator that uses order-inducing variables in the reordering of the arguments. Its main advantage is that it can represent more complex situations because it can include a wide range of factors in the reordering process rather than simply consider the values of the arguments. Recently, several authors have developed different extensions and applications of the IOWA operator. For example, Merigó and Gil-Lafuente (2009) generalized it by using generalized and quasi-arithmetic means. Merigó, López-Jurado, Gracia, and Casanovas (2009) provided a more general formulation by using hybrid aggregations. Wu, Li, Li, and Duan (2009) developed a continuous geometric version. Tan and Chen (2010) suggested an extension by using the Choquet integral. Herrera-Viedma, Chiclana, Herrera, and Alonso (2007) developed an extension for group decision-making problems. Merigó and Casanovas (2009) introduced an application in decision-making with Dempster–Shafer theory. For further reading, see for example Chen and Chen (2003), Merigó (2008), Merigó & Casanovas (2010a, 2011a, 2011c, 2011d), Merigó, Gil-Lafuente, and Barcellos (2010), Wei (2010) and Yager (2003).

Recently, some authors (Torra, 1997; Torra & Narukawa, 2007, 2010; Wei, 2009; Xu, 2010; Xu & Da, 2003; Yager, 1998b) have tried to unify the OWA operator with the WA in the same formulation. It is worth mentioning the work developed by Torra (1997) with the introduction of the weighted OWA (WOWA) operator and the work of Xu and Da (2003) about the hybrid averaging (HA) operator. Both models arrived to a unification between the OWA and the WA because both concepts were included in the formulation as particular cases. However, as it has been studied by Merigó (2008), these models seem to be a partial unification but not a real one because they can unify them but they cannot consider how relevant these concepts are in the specific problem considered. For example, in some problems we may prefer to give more importance to the OWA operator because we believe that it

* Tel.: +34 93 402 19 62; fax: +34 93 403 98 82.
E-mail address: jmerigo@ub.edu

0957-4174/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2011.03.034

is more relevant and vice versa. To overcome this issue, Merigo (2008, 2009) has developed a new approach that unifies the OWA and the WA in the same formulation considering the degree of importance that each concept may have in the problem.

In this paper, we present a new approach that unifies the OWA operator with the WA when we assess the information with induced aggregation operators. We call it the induced ordered weighted averaging–weighted average (IOWAWA) operator. We could also refer to it as the induced WOWA (IWOWA) operator but we have not done so because in the literature there is another approach that already uses a similar name (Torra, 1997). The main advantage of this approach is that it unifies the IOWA and the WA taking into account the degree of importance that each concept has in the formulation. Thus, we are able to consider situations where we give more or less importance to the IOWA and the WA depending on our interests on the problem analyzed. Note that by using the IOWA we are considering complex reordering processes affected by different factors such as the degree of optimism, time pressure and psychological aspects. By using the WA we are considering the subjective probability of the decision-maker against the possibility that each state of nature will occur.

We also develop different families of IOWAWA operators. We see that the IOWA, the OWA, the WA and the arithmetic mean are particular cases of this general formulation. Moreover, we are also able to unify the arithmetic mean (or simple average) with the IOWA operator when the weights of the WA are equal, obtaining the arithmetic-IOWA. A similar result called the arithmetic-WA is also found with the WA. We study other families such as the step-IOWAWA, the median-IOWAWA, the centered-IOWAWA, the S-IOWAWA and the olympic-IOWAWA.

We also analyze the applicability of the new model and we see that it is very broad because all the studies that use the IOWA or the WA can be revised and extended by using this new framework. The reason for this is that both the IOWA and the WA are included in this formulation as special cases. Therefore, in the worst of the cases, we can always simplify the model to the classical approach included in the IOWAWA. Thus, this model is much more robust and complete because it includes the classical approach and a much deeper formulation. We present some theoretical examples in statistics about the applicability of the IOWAWA obtaining the variance-IOWAWA (Var-IOWAWA), covariance-IOWAWA (Cov-IOWAWA) and a simple linear regression model with the IOWA-WA. We also focus on a decision-making problem about political management where we use a multi-person analysis. We see that depending on the particular type of IOWAWA operator used, the results may be different leading to different decisions.

The rest of the paper is organized as follows. In Section 2, we briefly describe some preliminaries. Section 3 presents the IOWA-WA operator and Section 4 several families. In Section 5 we study the applicability of the new approach including several examples in statistics. Section 6 analyzes the applicability in multi-person decision-making and Section 7 provides an illustrative example. In Section 8 we end the paper summarizing the main conclusions.

2. Preliminaries

In this section, we review the OWA, the IOWA, the WA and the OWAWA operator.

2.1. The OWA operator

The OWA operator (Yager, 1988) is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. It can be defined as follows.

Definition 1. An OWA operator of dimension n is a mapping $OWA: R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1,\ldots,a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest of the a_i.

Note that different properties could be studied such as the distinction between descending and ascending orders, different measures for characterizing the weighting vector and different families of OWA operators (Merigó, 2008; Yager, 1988, 1993; Yager & Kacprzyk, 1997; Yager, Kacprzyk, & Beliakov, 2011).

A useful issue in order to understand the IOWAWA operator is the reordering process of the aggregation. Usually, the arguments are reordered according to an established weighting vector. However, we may also develop the reordering process by reordering the weighting vector according to the positions of the arguments (Yager, 1998a). That is:

$$OWA(a_1,\ldots,a_n) = \sum_{i=1}^{n} a_i w_i, \tag{2}$$

where w_i is the ith weight w_j reordered according to the positions of the a_i.

2.2. The IOWA operator

The IOWA operator was introduced by Yager and Filev (1999) and it represents an extension of the OWA operator. Its main difference is that the reordering step is not developed with the values of the arguments a_i. In this case, the reordering step is developed with order inducing variables. The IOWA operator also includes as particular cases the maximum, the minimum and the average criteria. It can be defined as follows.

Definition 2. An IOWA operator of dimension n is a mapping $IOWA: R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, such that:

$$IOWA(\langle u_1,a_1\rangle,\langle u_2,a_2\rangle,\ldots,\langle u_n,a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the a_i value of the IOWA pair $\langle u_i, a_i\rangle$ having the jth largest u_i, u_i is the order-inducing variable and a_i is the argument variable.

Note that it is possible to distinguish between the descending IOWA (DIOWA) operator and the ascending IOWA (AIOWA) operator. The IOWA operator is also monotonic, bounded, idempotent and commutative. For further reading on the IOWA, refer, e.g., to Merigó (2008), Merigó and Gil-Lafuente (2009, in press), Wei (2010) and Yager (2003).

A key feature in order to understand the IOWAWA operator is the reordering process of the information. Usually, we reorder the IOWA according to the values of the u_i, but it is also possible to adapt them to the initial positions of the arguments. That is,

$$IOWA(\langle u_1,a_1\rangle,\langle u_2,a_2\rangle,\ldots,\langle u_n,a_n\rangle) = \sum_{i=1}^{n} a_i w_i, \tag{4}$$

where w_i is the ith weight w_j reordered according to the positions of the a_i and using order-inducing variables u_i.

Example 1. Assume we have the following arguments $A = (6, 80\rangle, 2, 30\rangle, 4, 20\rangle, 7, 50\rangle)$ and the following weighting vector $W = (0.4, 0.3, 0.2, 0.1)$. By using the IOWA operator (Eq. (3)) we get:

$$IOWA = 0.4 \times 50 + 0.3 \times 80 + 0.2 \times 20 + 0.1 \times 30 = 51,$$

and if we use Eq.(4), we obtain the following:

$$IOWA = 80 \times 0.2 + 30 \times 0.1 + 20 \times 0.2 + 50 \times 0.4 = 51.$$