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A goal programming-TOPSIS approach to multiple response optimization using the concepts of non-dominated solutions and prediction intervals

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ABSTRACT

Multiple response problems include three stages: data gathering, modeling and optimization. Most approaches to multiple response optimization ignore the effects of the modeling stage; the model is taken as given and subjected to multi-objective optimization. Moreover, these approaches use subjective methods for the trade off between responses to obtain one or more solutions. In contradistinction, in this paper we use the Prediction Intervals (PIs) from the model building stage to trade off between responses in an objective manner. Our new method combines concepts from the goal programming approach with normalization based on negative and positive ideal solutions as well as the use of prediction intervals for obtaining a set of non-dominated and efficient solutions. Then, the non-dominated solutions (alternatives) are ranked by the TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution) approach. Since some suggested settings of the input variables may not be possible in practice or may lead to unstable operating conditions, this ranking can be extremely helpful to Decision Makers (DMs). The consideration of statistical results together with the selection of the preferred solution among the efficient solutions by Multiple Attribute Decision Making (MADM) distinguishes our approach from others in the literature. We also show, through a numerical example, how the solutions of other methods can be obtained by modifying the relevant approach according to the DM's requirements.

1. Introduction

The control of production processes in an industrial environment requires the selection and correct setting of the input variables, so that on-specification product is produced at minimum cost. But first, the relationship between input and output variables must be determined. The series of techniques used in the empirical study of the association between response variables and several input variables is called Response Surface Methodology (RSM). See, for example, (Box & Draper, 1987 or Myers & Montgomery, 2002).

Much of the emphasis in RSM has been on building models for one response, whereas industrial processes often have many responses, the values of which ideally require simultaneous optimization. Usually optimizing all responses simultaneously would cause conflicts of interest. Our method of resolution of this conflict is based on realizing that the problem has three stages: data gathering, model building and optimization. At first the data are collected, using an experimental design. Then the techniques of RSM are applied for estimating the relation between response (output) and explanatory (input) variables and the model is constructed. Finally, the model is optimized. At this stage the purpose is to obtain optimum conditions on the input variables so that all responses concurrently will be as near as possible to their optima. Khuri (1996) discusses such multiple response problems.

Most research in multi-response optimization ignores the statistical uncertainty in the results of the modeling stage; the model is taken as given and subjected to multi-objective optimization. When conflicts exist between optimization of the various responses, subjective methods are typically used for trade off between responses to obtain a variety of alternative solutions. On the contrary, in our paper we use the Prediction Intervals (PIs) from the model building stage to trade off between responses in an objective manner. We then find the non-dominated solutions of the problem by modified goal programming. An advantage of our method is that the continuous region of solutions is transformed into a discrete region. Using the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) we rank the non-dominated solutions from this discrete region. This automatic procedure sometimes suggests impractical settings of the explanatory variables. But the ranking is a powerful tool for Decision Makers (DMs) who may need to modify the solutions for practical purposes. Our approach is distinguished from others in this field by

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the incorporation of statistical properties and by the selection of the preferred solution among the efficient solutions by Multiple Attribute Decision Making (MADM).

The structure of the paper is as follows. Section 2 reviews the existing work on multiple response problems and provides definitions relevant to the proposed method. The details of the method are expounded in Section 3. Section 4 clarifies the method by a numerical example. Finally, implementation issues are discussed in Section 5 with some conclusions in Section 6.

2. Literature review

This section first reviews some work on multi-response optimization (MRO). Then we present definitions necessary for the description of our proposed method.

2.1. A summary of existing work on MRO

The majority of approaches to the multi-response optimization problem in the literature fall in one of the following categories:

The first approach, loss function, considers three properties: bias, robustness and the quality of prediction. Bias measures systematic deviation, robustness refers to the low sensitivity of the response variable to nuisance factors and quality of prediction measures the variance of predictions. Originally Taguchi, Elsayed, and Hsiang (1989) presented a univariate loss function which ensured that the response converged to the target value with small variance. Pignatiello (1993) expanded Taguchi's method to a loss function for multi-response problems that considered both bias and robustness. Vining (1998) introduced a loss function that considered bias and quality. Finally, Ko, Kim, and Jun (2005) proposed a new loss function embracing all three properties.

The second approach, Multiple Objective Optimization (MOO), can itself be divided into three classes depending on the form of the preference information from the decision maker: prior, progressive, or posterior articulation methods (Hwang, Masud, Paidy, & Yoon, 1979). The prior method takes all of the preference information from the DM before solving the problem (e.g. Kazemzadeh, Bashiri, Atkinson, & Noorossana, 2008 who use goal programming to find the optimal setting of the controllable variables). In the progressive method, the solver and DM are in contact so that the solution is obtained interactively. As an example, Jeong and Kim (2009) proposed an interactive desirability function approach (IDFA) in which the shape, bound and target of the desirability function is subjectively changed. Further papers using interactive methods include (Köksalan & Plante, 2003; Mollaghasemi & Evans, 1994; Park & Kim, 2005). The posterior method finds all (or most) efficient solutions and then allows the DM to select the best one from the efficient solutions. We have failed to find any references to work employing the posterior method in MRO. Also in this category are the methods of Köksoy (2006) and of Köksoya and Yalcinozb (2006) which incorporate statistics such as Mean Squared Error (MSE) for robust design. For more details about MOO (see Collette & Siarry, 2003; Figueira, Greco, & Ehrgott, 2005). A summary review of MRO methods is in Table 1.

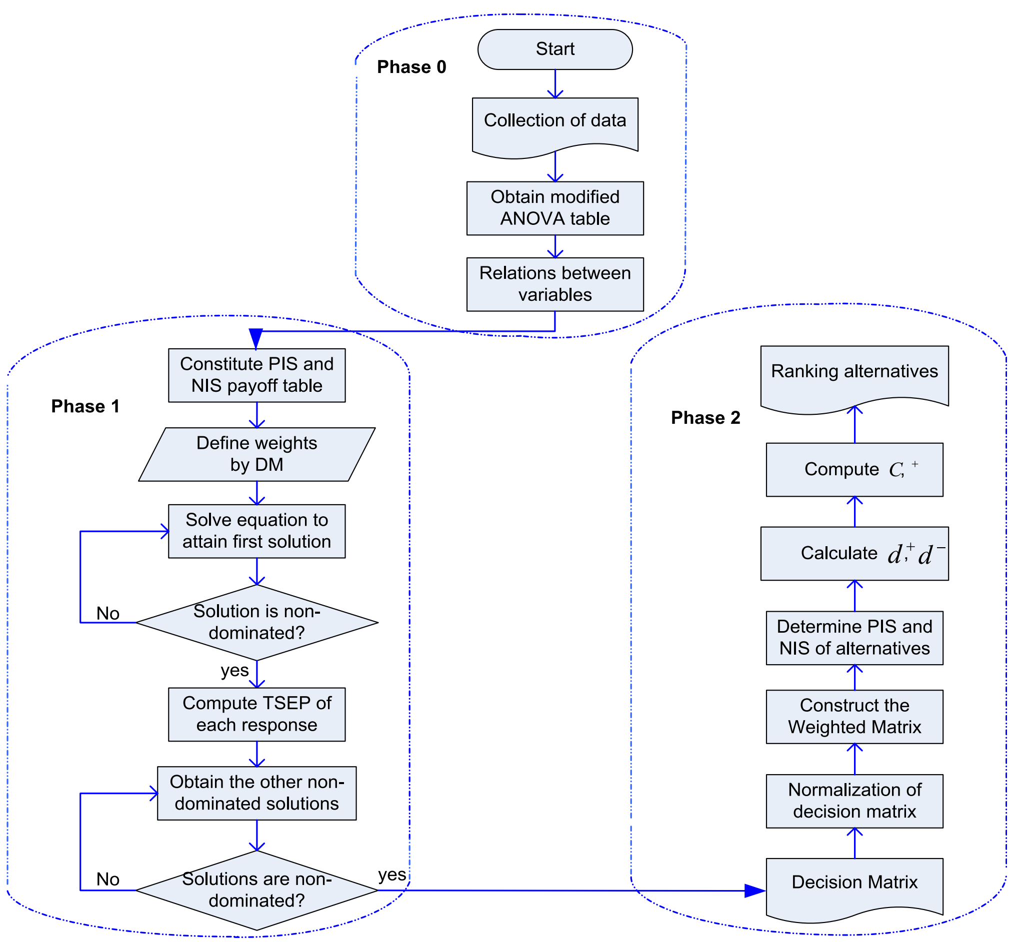

Fig. 1. The procedure of proposed method.