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Process capability analyses based on fuzzy measurements and fuzzy control charts

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ABSTRACT

Process performance can be analyzed by using process capability indices (PCIs), which are summary statistics to depict the process location and dispersion successfully. Although they are very usable statistics, they have some limitations which prevent a deep and flexible analysis because of the crisp measurements and specification limits (SLs). If the specification limits or measurements are expressed by linguistic variables, traditional PCIs cause some misleading results. In this paper, the fuzzy set theory is used to add more information and flexibility to process capability analyses (PCA). For this aim, linguistic definition of the quality characteristic measurements are converted to fuzzy numbers and fuzzy PCIs are produced based on these measurements and fuzzy specification limits (SLs). Also fuzzy control charts are derived for fuzzy measurements of the related quality characteristic. They are used to increase the accuracy of PCA by determining whether or not the process is in statistical control. The fuzzy formulation of the indices C_p and C_{pk}, which are the most used two traditional PCIs, are produced when SLs and measurements are both triangular (TFN) and trapezoidal fuzzy numbers (TrFN). The proposed methodologies are applied in a piston manufacturer in Konya's Industrial Area, Turkey.

1. Introduction

Statistical process control (SPC) is a large class of methods aiming at evaluating, monitoring and possibly reducing variability in industrial production processes and it plays an important role to assure the process is in statistical control. The two of the SPC functions are control charts and PCA. The control charts are used to monitor whether or not the process is in statistical control. Control charts are based on data representing one or several quality-related characteristics of the product or service. If these characteristics are measurable on numerical scales, then variable control charts are used. If the quality-related characteristics cannot be easily represented in numerical form, then attribute control charts are useful. Generally, they concentrate on if the process is "in control" or "out of control". If the process is in control, evaluation and forecasting of the process capability has a meaning.

The other SPC tool, process capability analysis (PCA), can be broadly defined as the ability of a process to meet customer expectations which are defined as specification limits (SLs). The measure of process capability summarizes some aspects of a process's ability to meet SLs and it is a very useful approach to define a relationship between the process ability and SLs. The main outputs of PCA are process capability indices (PCIs) which provide a numerical measure of whether a production process is capable of producing items within the specification limits predetermined by the designer or consumer. If the certain minimum values of PCIs have been obtained the process is classified as "capable process" which meaning that it have a success for meeting SLs. If these minimum values cannot be met, the process is classified as "incapable process". The larger process capability index implies the higher process yield, and the larger process capability index also indicates the lower process expected loss. Therefore, the process capability index can be viewed as an effective and excellent means of measuring product quality and performance. Stability or statistical control of the process is also essential to the correct interpretation of PCIs. If the process is not in control, then of course its parameters are unstable, and the value of these parameters in the future is uncertain (Montgomery, 2005). If a process is out of control, it has historically failed to display a reasonable degree of consistency and it is illogical to expect that it will spontaneously do so in the future. Therefore, a capability study monitoring an unstable process will only express the capability of the process at that very moment and nothing can be said about the capability of the process in the future (Deleryd, 1998). We know that the best tool to check whether or not the process is in statistical control is control charts. In this paper, the process is firstly checked to determine whether or not it is in statistical control. Then the PCA is managed for the quality characteristics.

Most of the conventional studies for measuring the process capability are based on crisp estimates wherein the output process measurements are precise. However, it is not common that the measurements of product quality are insufficiently precise.

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Uncertainty arises in real applications from sources such as estimation from sample data; observations with coarse scales; measurement error that is not quantified accurately; and imprecise specification limits. Real observations of continuous quantities are not precise numbers – they are more or less imprecise. The best description of such data is by so-called imprecise numbers. Such observations are also called fuzzy. A typical example for an imprecise number is the lifetime of a system which cannot, in general, be described by one real number because the time at which the lifetime ends is not a precise number but is more or less imprecise. Other examples of imprecise data are the data given by color intensity pictures or readings on analogue measurement equipments. Also readings on digital measurement equipments are not precise numbers but intervals since there are only a finite number of decimals available (Wu, 2009). Thus, PCIs cannot be defined exactly or represented by a crisp value precisely. Fuzzy logic is a branch of mathematics that allows a computer to model the real world in the same way that people do. It provides a simple way to reason with vague, ambiguous, and imprecise input or knowledge.

In this paper, PCIs are obtained for fuzzy measurements which are defined by linguistic or approximate values. To check if a statistical control exists, fuzzy control charts are produced and then fuzzy PCIs are derived. The rest of this paper is organized as follows: Traditional PCIs are briefly summarized in Section 2. Section 3 includes the main principles of fuzzy control charts and some rules to check the process situation. Also fuzzy PCIs are introduced in this section. Section 4 includes an application which is proceeded by a computer program coded in Visual Basic. Conclusions, main advantages of FPCIs and future research directions are discussed in Section 5.

2. Traditional process capability indices

The PCA compares the output of a process to SLs by using process capability indices (PCIs). This comparison is made by forming the ratio of the width between the process specification limits to the width of the natural tolerance limits which is measured by six process standard deviation units (Montgomery, 2005). In the literature some PCIs such as C_p, C_{pk}, C_{pm}, C_{pmk}, and C_a have been used to measure the ability of process to decide how well the process meets the specification limits. The important steps and decisions involved with conducting a process capability study are diagrammed on the flowchart in Fig. 1.

As seen in Fig. 1 clearly, before a process can be said to have a well defined capability, it must be in statistical control, which basically means that the process is predictable within reasonable limits.

Process capability measures are determined under the assumption that the process is in statistical control which means that the variation is due only to random causes. Regardless of the frequency or magnitude of the out-of-control signal, traditional process capability measures cannot be used in the presence of variation due to an assignable cause (i.e. process is not in-control). Control charts are designed to identify variation due to assignable causes and will play an important role in the proposed strategy. Control charts will be used to provide evidence that the process is in-control. If the process is in-control, then the values of PCIs provide information regarding process capability. An out-of-control signal may arise in several fashions and practitioners are warned not to calculate PCIs at these and subsequent points. Once appropriate actions have been taken to deal with the assignable cause and the process is returned to a state of statistical control, PCIs can again be calculated. The usual control chart procedures are used first to verify the assumption that the process is in-control. If the process is deemed in-control, then estimates of the process capability can be calculated from the subgroup information (Spiring, 1995). Non-random patterns on a control chart suggest the presence of assignable-cause variations that will make the result of any capability index meaningless (Xie, Tsui, Goh, & Cai, 2002). In this paper, fuzzy control charts are derived to check whether or not the process is in statistical control for fuzzy measurements of quality characteristics. Then fuzzy PCIs are produced.

There have been a number of process capability indices proposed over the years for the purpose of assessing the capability of a process to meet certain specifications. The two most widely used standard PCIs are C_p and C_{pk}. The index C_p which is the first process capability index (PCI) to appear in the literature and called precision index (Kane, 1986) is defined as the ratio of specification width ($USL - LSL$) over the process spread (6σ). The specification width represents customer and/or product requirements. The process variations are represented by the specification width. If the process variation is very large, the C_p value is small and it represents a low process capability. C_p indicates how well the process fits within the two specification limits. It never considers any process shift as presented in Fig. 2 and it is calculated by using Eq. (1). C_p simply measures the spread of the specifications relative to the six-sigma spread in the process (Kotz & Johnson, 2002; Montgomery, 2005).

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where σ is the standard deviation of the process. USL and LSL are upper and lower specification limits, respectively.

The value of index C_p gives us an opinion about process' performance. For example if it is greater than $1.3\overline{3}$ which corresponds to 63 nonconforming parts per million (ppm) for a centered process, we conclude that process performance is satisfactory. The six quality conditions and the corresponding $C_p$values are summarized in Table 1 (Tsai & Chen, 2006).

The process capability ratio C_p does not take into account where the process mean is located relative to specifications (Montgomery, 2005). C_p focuses on the dispersion of the studied process and does not take into account centering the process and thus gives no indication of the actual process performance. Kane (1986) introduced index C_{pk} to overcome this problem. The index C_{pk} is used to provide an indication of the variability associated with a process. It shows how a process confirms to its specifications. The index is usually used to relate the "natural tolerances (3σ)" to the specification limits. C_{pk} describes how well the process fits within the specification limits, taking into account the location of the process mean. C_{pk} should be calculated based on Eqs. (2)–(4) (Kane, 1986; Kotz & Johnson, 2002; Montgomery, 2005).

$$C_{pk} = \min\{C_{pl}, C_{pu}\} \tag{2}$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \tag{3}$$

$$C_{pu} = \frac{USL - \mu}{3\sigma} \tag{4}$$

C_p compares the 6σ spread of the process with the tolerance spread. It does not require knowledge of the process location μ and for this reason it can be seen as a measure for the process capability of an optimally centered process. C_{pk} was introduced to give μ some influence on the value of the index. Both C_p and C_{pk} are designed to reflect changes in the amount of product beyond the specification limits.

3. Fuzzy process capability indices and control charts

After the inception of the notion of fuzzy sets by Zadeh (1965), many authors have applied this approach to very different areas