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Evaluating process performance based on the incapability index for measurements with uncertainty

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ABSTRACT

Process capability indices are widely used in industry to measure the ability of firms or their suppliers to meet quality specifications. The index C_{PP}, which is easy to use and analytically tractable, has been successfully developed and applied by competitive firms to dominate highly-profitable markets by improving quality and productivity. Hypothesis testing is very essential for practical decision-making. Generally, the underlying data are assumed to be precise numbers, but in general it is much more realistic to consider fuzzy values, which are imprecise numbers. In this case, the test statistic also yields an imprecise number, and decision rules based on the crisp-based approach are inappropriate. This study investigates the situation of uncertain or imprecise product quality measurements. A set of confidence intervals for sample mean and variance is used to produce triangular fuzzy numbers for estimating the C_{PP} index. Based on the δ-cuts of the fuzzy estimators, a decision testing rule and procedure are developed to evaluate process performance based on critical values and fuzzy *p*-values. An efficient computer program is also designed for calculating fuzzy *p*-values. Finally, an example is examined for demonstrating the application of the proposed approach.

1. Introduction

Process capability is a critical performance measure addressing process results with regard to product specifications. Process capability indices thus are widely used in industry to measure the ability of firms or their suppliers to meet quality specifications. Greenwich and Jahr-Schaffrath (1995) introduced an "incapability" index called C_{PP}, which is easy to use and analytically tractable. The formulae of C_{PP} is easy to understand and apply, but in practice the parameters, process mean μ and process standard deviation σ, of C_{PP} are generally unknown, and the C_{PP} value must be estimated based on a random sample $X_1, X_2, \ldots, X_n$. Chen (1998) obtained the probability density function (PDF) and the *r*th moments of the uniformly minimum variance unbiased estimator (UMVUE) of C_{PP}. Furthermore, Pearn and Lin (2001) investigated the statistical properties of the estimated C_{PP} and derived the upper confidence limit of C_{PP}. A decision-making procedure is also devised for assessing whether the process satisfies the preset quality requirement. Lin and Pearn (2005) presented an efficient SAS computer program for calculating the *p*-value and critical value for hypothesis testing. Furthermore, Lin (2007) obtained the posterior probability of which the investigated process is capable based on C_{PP} using the Bayesian approach. Additionally, Pearn, Ko, and Wang (2002) presented a C_{PP} multiple process performance analysis chart (MPPAC) for processes possessing multiple independent characteristics. Moreover, Chen, Chen, and Li (2005) defined a price index, and constructed a supplier capability and price analysis chart (SCPAC) based on the price index and the index C_{PP} for supplier evaluation.

Most studies on process capability analysis are based on crisp estimates involving precise output process measurements. However, measurements of product quality sometimes cannot be precisely recorded or collected, making imprecise or fuzzy numbers the only feasible means of describing such data. Since measures of product quality often lack precision, a new trend has been inspired of combining randomness and fuzziness in assessing process capability. Yongting (1996) first proposed the fuzzy attribute of quality, a formula of fuzzy process capability index $\underset{\sim}{C}_P$, which can be defined as the probability of fuzzy up-to-standard products produced by production processes, is proposed for dealing with fuzzy processes. Lee (2001) and Hong (2004) obtained the membership functions of the mean and standard deviation of fuzzy numbers, then conducted the C_{PK} index estimation presented by fuzzy number and approximated the membership function of the C_{PK} index. Chen, Chen, and Lin (2003) incorporated the fuzzy inference using index C_{PL} for processes with bigger-the-better type quality characteristics, and employed a concise score concept to represent the grade of process capability. Furthermore, Tsai and Chen (2006) considered the applications of index C_P in the fuzzy environment, and formulated a pair of nonlinear functions to

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identify the approximate membership function. Additionally, Parchami, Mashinchi, Yavari, and Maleki (2005) and Parchami and Mashinchi (2007) introduced fuzzy process capability indices and discussed the relationships between them in the cases where the specification limits are triangular fuzzy numbers rather than crisp numbers.

There are few studies on assessing process capability based on C_{PP} with consideration of measurements with uncertainty. Recently, Chen and Chen (2008) assessed multiprocess capability using distance value of a confidence box. A fuzzy inference method was proposed for determining the distance value, and this approach can determine the optimal process. Although their approach compares capability among processes, it cannot be applied to determine single process capability for process with fuzzy data. To overcome this problem, this study describes a simple but practical method by an extended version of the approach of Buckley and Eslami (2004), Buckley (2004, 2005). A set of confidence intervals of the sample mean and variance is established to yield triangular fuzzy numbers for estimating C_{PP}. Moreover, a three-decision testing rule and a step-by-step procedure are developed to assess process performance using critical value and fuzzy p-value. The three-decision testing rule can be considered a natural generalization of the traditional crisp-based test, and can be reduced to the traditional process capability test by binary decision-making in situations involving precise data. The remainder of this paper is organized as follows. Section 2 briefly discusses the crisp estimation for the index C_{PP}. Next, Section 3 introduces the δ-cuts of fuzzy estimation for C_{PP}. Section 4 then proposes the decision rules and testing procedures based on critical value and fuzzy p-value for assessing process capability, and provides an R computer program to calculate fuzzy p-values. To illustrate the applicability of the proposed approach, an example is presented in Section 5. Finally, conclusions are given in the last section.

2. The process incapability index C_{PP} and its crisp estimation

This section introduces the index C_{PP}, and then briefly discusses the statistical properties of the estimator of C_{PP} for crisp data.

2.1. The index C_{PP}

The index C_{PP} is defined as

$$C_{PP}=\left(\frac{\mu-T}{D}\right)^2+\left(\frac{\sigma}{D}\right)^2, \tag{1}$$

where μ is the process mean, σ is the process standard deviation, USL is the upper specification limit, LSL is the lower specification limit, $T=(USL+LSL)/2$ is the target value, and $D=(USL-LSL)/6$. Let $C_{ia}=(\mu-T)^2/D^2$ and $C_{iP}=\sigma^2/D^2, C_{PP}$ can be expressed as $C_{PP}=C_{ia}+C_{iP}$. The index C_{ia} measures the relative process departure, which reflects process inaccuracy. The index C_{iP} measures the process variation relative the specification tolerance, which reflects process imprecision. Thus, C_{PP} provides an uncontaminated separation between information concerning process accuracy and process precision.

Process yield, the percentage of processed product units passing inspection, is a standard numerical measure of process performance in manufacturing. The expected process yield of given values of C_{ia} and C_{iP} is

$$\begin{aligned}\%\text{yield}&=\Phi\left(\frac{USL-\mu}{\sigma}\right)+\Phi\left(\frac{\mu-LSL}{\sigma}\right)-1\\&=\Phi\left(\frac{3+\sqrt{C_{ia}}}{\sqrt{C_{iP}}}\right)+\Phi\left(\frac{3-\sqrt{C_{ia}}}{\sqrt{C_{iP}}}\right)-1,\end{aligned} \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution $N(0,1)$. We obtain that the C_{PP} value is increasing by C_{ia} or C_{iP}, and the process yield is decreasing by C_{ia} or C_{iP}. Thus, it indicates that increment of process departure or process variation would result in a larger C_{PP} value and diminish the process yield.

The C_{PP} value can directly reflect the process incapability. In general, a process is called "inadequate" if $C_{PP}>1.00$, called "marginally capable" if $0.57<C_{PP}\leqslant 1.00$, called "capable" if $0.44<C_{PP}\leqslant 0.57$, called "good" if $0.36<C_{PP}\leqslant 0.44$, called "excellent" if $0.25<C_{PP}\leqslant 0.36$, and is called "super" if $C_{PP}\leqslant 0.25$. Contrary to other process capability indices, process capability decreases with increasing value of C_{PP}, which is why C_{PP} is called a process incapability index. Furthermore, C_{PP} provides more information regarding the process, including process inaccuracy and imprecision, than other indices, thus helping better understand the process situation of the contract manufactures to improve quality performance.

2.2. The estimator of C_{PP}

To estimate the yield measurement index C_{PP}, Pearn and Lin (2001) consider the following estimator $\widehat{C}_{PP}$,

$$\widehat{C}_{PP}=\left(\frac{\bar{x}-T}{D}\right)^2+\left(\frac{s_n}{D}\right)^2, \tag{3}$$

where $\bar{x}=\sum_{i=1}^{n}x_i/n$, and $s_n=[\sum_{i=1}^{n}(x_i-\bar{x})^2/n]^{1/2}$ are maximum likelihood estimators (MLEs) of μ and σ, respectively, which may be obtained from a stable process. The estimator $\widehat{C}_{PP}$ can be rewritten as

$$\begin{aligned}\widehat{C}_{PP}&=\left(\frac{C_{iP}}{n}\right)\left(\frac{nD^2}{\sigma^2}\right)\left(\sum_{i=1}^{n}\frac{(x_i-\bar{x})^2}{nD^2}+\frac{(\bar{x}-T)^2}{D^2}\right)\\&=\left(\frac{C_{iP}}{n}\right)\left(\sum_{i=1}^{n}\frac{(x_i-T)^2}{\sigma^2}\right).\end{aligned} \tag{4}$$

Since $\sum_{i=1}^{n}(x_i-T)^2/\sigma^2$ is distributed $\chi_n^2(\lambda)$, a non-central chi-square distribution with n degrees of freedom and non-centrality parameter $\lambda=n(\mu-T)^2/\sigma^2=nC_{ia}/C_{iP}$, the estimator $\widehat{C}_{PP}$ is distributed as $(C_{iP}/n)\chi_n^2(\lambda)$. The PDF of $\widehat{C}_{PP}$ can be expressed as

$$f_{\widehat{C}_{PP}}(y)=\sum_{j=0}^{\infty}\left\{\frac{[ny/2C_{iP}]^{j+(n/2)}\exp[-ny/2C_{iP}]}{y\Gamma[j+(n/2)]}\times\frac{(\lambda/2)^j\exp(-\lambda/2)}{\Gamma(j+1)}\right\} \tag{5}$$

for $y>0$. And the rth moment of $\widehat{C}_{PP}$ is

$$E(\widehat{C}_{PP}^r)=\sum_{j=0}^{\infty}\left\{\left(\frac{2C_{iP}}{n}\right)^r\frac{\Gamma[j+(n/2)+r]}{\Gamma[j+(n/2)]}\times\frac{(\lambda/2)^j\exp(-\lambda/2)}{\Gamma(j+1)}\right\}. \tag{6}$$

Pearn and Lin (2001) showed that $\widehat{C}_{PP}$ is the MLE, which is also the UMVUE of C_{PP}. They also showed that $\widehat{C}_{PP}$ is consistent, $\sqrt{n}(\widehat{C}_{PP}-C_{PP})$ converges to $N(0,2C_{iP}C_{ia}+2C_{iP}C_{PP})$, and $\widehat{C}_{PP}$ is asymptotically efficient. The $100(1-\alpha)\%$ upper confidence bound, U, for C_{PP} can be solved by the following equation (Pearn & Lin, 2001),

$$\begin{aligned}1-\alpha&=P(C_{PP}<U)=P(C_{PP}-C_{ia}<U-C_{ia})\\&=P\left(\frac{1}{C_{PP}-C_{ia}}>\frac{1}{U-C_{ia}}\right)=P\left(\frac{n\widehat{C}_{PP}}{C_{PP}-C_{ia}}>\frac{n\widehat{C}_{PP}}{U-C_{ia}}\right)\\&=P\left(\chi_n^2(\lambda)>\frac{n\widehat{C}_{PP}}{U-C_{ia}}\right).\end{aligned} \tag{7}$$

Therefore, $U=C_{ia}+n\widehat{C}_{PP}/\chi_{1-\alpha,n}^2(\lambda)$, where $\chi_{\alpha,n}^2(\lambda)$ is the upper αth percentile of $\chi_n^2(\lambda)$ distribution. Since μ and σ^2 are unknown, we can use $\bar{x}$ and s_n^2 to estimate μ and σ^2 in practical applications.