Expert Systems with Applications 38 (2011) 121–130

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Delay dependent stability results for fuzzy BAM neural networks with Markovian jumping parameters

P. Balasubramaniam *, R. Rakkiyappan, R. Sathy

Department of Mathematics, Gandhigram Rural University, Gandhigram 624 302, Tamilnadu, India

ARTICLE INFO

Keywords:
BAM neural networks
Fuzzy systems
Linear matrix inequality
Lyapunov–Krasovskii functional
Markovian jumping parameters

ABSTRACT

This paper deals with the delay-dependent asymptotic stability analysis problem for a class of fuzzy bidirectional associative memory (BAM) neural networks with time-varying interval delays and Markovian jumping parameters by Takagi–Sugeno (T–S) fuzzy model. The nonlinear delayed BAM neural networks are first established as a modified T–S fuzzy model in which the consequent parts are composed of a set of Markovian jumping BAM neural networks with time-varying interval delays. The jumping parameters considered here are generated from a continuous-time discrete-state homogeneous Markov process, which are governed by a Markov process with discrete and finite-state space. The new type of Markovian jumping matrices P_k and Q_k are introduced in this paper. The parameter uncertainties are assumed to be norm bounded and the delay is assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. A new delay-dependent stability condition is derived in terms of linear matrix inequality by constructing a new Lyapunov–Krasovskii functional and introducing some free-weighting matrices. Numerical examples are given to demonstrate the effectiveness of the proposed methods.

1. Introduction

It is well known that the dynamics of neural networks such as cellular neural networks (CNNs) (Cao & Wang, 2005), Hopfield neural networks (HNNs) (Hopfield, 1982) and bidirectional associative memory (BAM) (Kosko, 1988) have been deeply investigated in recent years due to its applicability in solving some image processing, signal processing, optimization, pattern recognition problems, fixed point computations and other areas. In practice, time delays often occur in many dynamic systems, such as rolling and biological systems, metallurgical processes, network systems, and so on. In the past decade, stability analysis and synthesis have been addressed extensively for time-delay systems, and a large amount of research results has been reported in the literature (Chen, Lam, & Xu, 2006; Liu, Han, & Li, 2009; Shen & Wang, 2008; Sun, Wang, & Zhao, 2008; Sun, Zhao, & Hill, 2006; Wang & Zhao, 2007). The existing results can be classified into two types: delay-independent criteria (Arik, 2004) and delay-dependent criteria (Liao, Liu, & Zhang, 2006; Zhang, Wei, & Xu, 2007), and references therein. The former is irrespective of the size of the delay and the later is concerned with the size of the delay. It has been shown that the delay-dependent stability conditions are generally less conservative than the delay-independent ones, especially when the size of the delay is small. The stability analysis of BAM neural networks with delays has attracted considerable interest, see, for example Chen, Huang, Liu, and Cao (2006), Rao and Phaneendra (1999) and references therein.

Hybrid systems driven by continuous-time Markov chain have been used to model many practical systems, where they may experience abrupt changes in their structure and parameters (Lou & Cui, 2007b; Mao, 2002; Sworder & Rogers, 1983; Willsky & Rogers, 1979). Stochastic neural network with Markovian jumping parameters is one of such hybrid systems, where the parameters are governed by a discrete-state homogeneous Markov process, and every state denotes a switching mode. For Markovian switching neural networks, there are some developments in the recent years. For example Wang, Liu, Yu, and Liu (2006) studied the exponential stability of neural networks with discrete time-invariant delays, Huang, Ho, and Qu (2007) considered a stochastic neural network with discrete time-delays and parameter uncertainties. Recently, linear matrix inequality (LMI)-based stochastic exponential stability was proposed in Lou and Cui (2007a) for Markovian jumping BAM neural networks with time-varying delays. In Lou and Cui (2007b) and Li, Chen, Zhou, and Lin (2008), the problem of delay-dependent stochastic stability for a class of time-delay HNNs with Markovian jump parameters was considered.

Among various methods developed for the analysis and synthesis of complex nonlinear systems, fuzzy logic control is an attractive and effective rule-based one. In many of the model-based fuzzy control

* Corresponding author. Tel.: +91 451 2452371; fax: +91 451 2453071.
E-mail address: pbalgri@rediffmail.com (P. Balasubramaniam).

0957-4174/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2010.06.025

approaches, the well-known T–S fuzzy model (Takagi & Sugeno, 1985) is a popular and convenient tool in functional approximations. During the last decades, considerable attention has been paid to the stability analysis and control synthesis of T–S fuzzy systems (Chen, Liu, & Tong, 2007; Gao & Chen, 2007; Lin, Wang, & Lee, 2006; Lin, Wang, Lee, He, & Chen, 2007; Tong & Li, 2002; Tong, Wang, & Qu, 2007). Recently, the T–S fuzzy model approach has been used to investigate nonlinear stochastic time-delay systems (Wang, Ho, & Liu, 2004; Zhang, Xu, Zong, & Zou, 2007) and nonlinear Markovian jump systems (Nguang, Assawinchaichote, Shi, & Shi, 2005a; Nguang, Assawinchaichote, Shi, & Shi, 2005b; Wu & Cai, 2006; Wu & Cai, 2007). In recent years, the concept of incorporating fuzzy logic into a neural network has grown into a popular research topic (Huang, 2006; Huang, Ho, & Lam, 2005; Li, Chen, Lin, & Zhou, 2009; Li, Chen, Zhou, & Qian, 2009; Liu & Shi, 2009; Lou & Cui, 2007b). In Lou and Cui (2007b), the global asymptotic stability problem of T–S fuzzy BAM neural networks with time-varying delays and parameter uncertainties is considered. In Lou and Cui (2007b), the generalized T–S fuzzy models can be used to represent some complex nonlinear systems by having a set of nonlinear delayed systems as its consequent parts (Cao & Frank, 2000). To the best of the authors' knowledge, the robust stability problem for uncertain fuzzy BAM neural networks with Markovian jumping and time-varying interval delays has not been fully investigated, which is very challenging and remains as an open issue.

In this paper, we contribute to the development of stability analysis of Markovian jumping fuzzy BAM neural networks with time-varying interval delays. The dynamical system under consideration consists of time-varying discrete delays without any restriction on upper bounds of derivatives of time-varying delays. A new type of Markovian jumping matrices P_k and Q_k is introduced to derive several sufficient conditions for delay-dependent stability analysis of fuzzy BAM neural networks. A feature of the reported results is that delay-range-dependent robust stability problem is investigated by constructing a Lyapunov functional including both lower and upper bounds of delay and utilizing the free-weighting matrix method (He, Wang, Lin, & Wu, 2007; He, Wu, She, & Liu, 2004). Another feature of the results lies in that a new method is proposed to estimate the upper bound of the derivative of Lyapunov functional without ignoring some useful integral terms. Delay-range-dependent robust stability conditions are presented in terms of LMIs, which can be readily verified by using standard numerical software. Numerical examples are given to illustrate the effectiveness of the proposed results.

The rest of this paper is organized as follows. Section 2 states the problem description and preliminaries. Section 3 includes the sufficient conditions for delay-dependent stability analysis and Section 4 provides delay-dependent robust stability criterion for the system. Section 5 provides illustrative examples and Section 6 concludes the paper.

Notation: Throughout this paper, for symmetric matrices X and Y, the notation $X \geqslant Y$ ($X > Y$) means that $X - Y$ is positive-semidefinite (positive-definite); M^T denotes the transpose of the matrix M; I is the identity matrix with appropriate dimension; $(\Omega, \mathscr{F}, \mathscr{P})$ is a probability space with Ω as the sample and $\mathscr{F}$ as the algebra of the subsets of the sample space; $\mathbb{E}(\cdot)$ stands for the expectation operator with respect to the given probability measure $\mathscr{P}$; and matrices, if not explicitly stated, are assumed to have compatible dimensions. "*" denotes a block that is readily inferred by symmetry.

2. Problem description and preliminaries

Given a probability space $(\Omega, \mathscr{F}, \mathscr{P}), \{\eta_t, t \geqslant 0\}$ is a homogeneous finite-state Markovian process with right continuous trajectories and taking values in finite set $S = \{1,2,\ldots,s\}$ with the initial model η_0. Let $\Pi = [\pi_{kk^*}], k, k^* \in S$ denotes the transition rate matrix with transition probability

$$\Pr(\eta_{t+\Delta t} = k'|\eta_t = k) = \begin{cases} \pi_{kk'}\Delta t + o(\Delta t), & k \neq k' \\ 1 + \pi_{kk}\Delta t + o(\Delta t), & k = k' \end{cases}$$

where $\Delta t > 0$, $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$, and $\pi_{kk'}$ is the transition rate from mode k to mode k', satisfying $\pi_{kk'} \geqslant 0$ for $k \neq k'$ with $\pi_{kk} = -\sum_{k'=1,k'\neq k}^{s} \pi_{kk'}, k, k' \in S$.

Consider the following fuzzy BAM neural networks with time-varying delays and Markovian jumping parameters as

$$\begin{cases} \dot{u}_i(t) = -a_i(\eta(t))u_i(t) + \sum\limits_{j=1}^{n} b_{ji}(\eta(t))F_j(v_j(t)) \\ \qquad + \sum\limits_{j=1}^{n} c_{ji}(\eta(t))F_j(v_j(t-\rho(t))) + I_i, \\ \dot{v}_j(t) = -d_j(\eta(t))v_j(t) + \sum\limits_{i=1}^{m} e_{ij}(\eta(t))G_i(u_i(t)) \\ \qquad + \sum\limits_{i=1}^{m} f_{ij}(\eta(t))G_i(u_i(t-\tau(t))) + \bar{I}_j \end{cases} \tag{1}$$

for $i = \{1,2,\ldots,m\}$, $j = \{1,2,\ldots,n\}$, $t > 0$, where $u_i(t)$ and $v_j(t)$ denote the activations of the ith neurons and jth neurons, respectively; $F_j(\cdot)$ and $G_i(\cdot)$ stand for the signal functions of the ith neurons and jth neurons, respectively; $a_i(\eta(t))$ and $d_j(\eta(t))$ are positive constants, they stand for the rate with which the cell i and j reset their potential to the resting state when isolated from the other cells and inputs: $b_{ji}(\eta(t))$, $c_{ji}(\eta(t))$, $e_{ij}(\eta(t))$ and $f_{ij}(\eta(t))$ denote the synaptic connection weights; I_i and $\bar{I}_j$ denote the external inputs at time t. The bounded function $\tau(t)$ and $\rho(t)$ represent unknown delays of systems and satisfy

$$h_1 \leqslant \tau(t) \leqslant h_2, \quad \dot{\tau}(t) \leqslant \mu, \tag{2}$$

$$\rho_1 \leqslant \rho(t) \leqslant \rho_2, \quad \dot{\rho}(t) \leqslant \eta. \tag{3}$$

(**A**) We assume that there exist positive w_i^1, w_j^2 such that

$$|F_j(x_1) - F_j(x_2)| \leqslant w_j^2|x_1 - x_2|, \quad |G_i(x_1) - G_i(x_2)| \leqslant w_i^1|x_1 - x_2| \tag{4}$$

for all $x_1, x_2 \in \mathbb{R}$; $x_1 \neq x_2$, where $w_i^1 > 0$, $w_j^2 > 0$ denote Lipschitz constant, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

The system (1) is supplemented with initial values given by

$$\begin{cases} u_i(t) = \phi_{ui}(t), & t \in [-h_2, 0], \quad i = 1,2,\ldots,m, \\ v_j(t) = \phi_{vj}(t), & t \in [-\rho_2, 0], \quad j = 1,2,\cdots,n, \end{cases}$$

where $\phi_{ui}(t)$, $\phi_{vj}(t)$ are continuous functions defined on $[-h_2, 0]$ and $[-\rho_2, 0]$, respectively.

The system (1) is equivalent to the vector form as follows:

$$\begin{cases} \dot{u}(t) = -A(\eta_t)u(t) + B(\eta_t)F(v(t)) + C(\eta_t)F(v(t-\rho(t))) + I, \\ \dot{v}(t) = -D(\eta_t)v(t) + E(\eta_t)G(u(t)) + F(\eta_t)G(u(t-\tau(t))) + \bar{I}, \end{cases} \tag{5}$$

where

$$u = (u_1, u_2, \ldots, u_m)^T, \quad v = (v_1, v_2, \ldots, v_n)^T,$$

$$A(\eta_t) = \mathrm{diag}(a_1(\eta_t), a_2(\eta_t), \ldots, a_m(\eta_t)),$$
$$D(\eta_t) = \mathrm{diag}(d_1(\eta_t), d_2(\eta_t), \ldots, d_n(\eta_t)),$$

$$B(\eta_t) = \left[(b_{ji}(\eta_t))_{n\times m}\right]^T, \quad C(\eta_t) = \left[(c_{ji}(\eta_t))_{n\times m}\right]^T,$$
$$I = (I_1, I_2, \ldots, I_m)^T,$$

$$E(\eta_t) = \left[(e_{ij}(\eta_t))_{m\times n}\right]^T, \quad F(\eta_t) = \left[(f_{ij}(\eta_t))_{m\times n}\right]^T, \quad \bar{I} = (\bar{I}_1, \bar{I}_2, \cdots, \bar{I}_n)^T,$$

and nonlinear active functions

$$F(v(t)) = F_j(v_j(t))_{n\times 1}, \quad F(v(t-\rho(t))) = F_j(v_j(t-\rho(t)))_{n\times 1},$$