Expert Systems with Applications 38 (2011) 564–570

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A DEA-inspired procedure for the aggregation of preferences

I. Contreras *

Department of Economy, Quantitative Methods and Economic History, Pablo de Olavide University, Spain

ARTICLE INFO

Keywords:
Group decision-making
Data Envelopment Analysis
Goal programming

ABSTRACT

Among the most commonly applied methods for the aggregation of individual preferences, weighted scoring rules associate each alternative with a weighted sum of votes received and then rank them in terms of this aggregate value. Some authors have argued that it is not possible to fairly evaluate a set of alternatives by only considering an externally imposed weight scheme. For this reason, researchers have developed certain procedures in which the weights associated with the votes become variables in the model. Data Envelopment Analysis (DEA) represents one class of such models.

In this paper, I propose a new preference-aggregation procedure. The procedure maintains the philosophy inherent in DEA, allowing each alternative to have its own vector of weights, but also introduces a new objective in the evaluation, the optimisation of the rank position in which the alternative is placed, and to avoid the problem of diverse weights by determining a social ranking that uses a common vector of weights.

1. Introduction

The manner in which a group or social ranking from a set of individual preferences is achieved is an important aspect of the decision-making context. This class of procedures contains either those in which each decision maker (DM) expresses his or her preferences in terms of a rank order of alternatives or those in which each DM is asked to select a subset of alternatives from a feasible set and then rank them from least to most preferred. Among the procedures developed for these situations, one well-known family of methods is weighted scoring rules.

Weighting scoring rules operate by computing, for each alternative, a score that depends on the rank position of the alternative in the individual's order of preferences. Subsequently, the alternatives are ranked by the sum of scores received. The value obtained by the ith alternative is computed as $V_i = \sum_j w_i v_{ij}$, where v_{ij} denotes the number of jth-place ranks that alternative i occupies and w_j is the scoring or weighting applied to the rank-position votes. Because a number is assigned to each candidate, these procedures guarantee a weak ordering of alternatives.

The Borda–Kendall rule is considered the origin of this class of preference-aggregation procedures. Among the most widely discussed scoring rules are (leaving the Borda–Kendall rule aside) the plurality rule, the antiplurality rule and other collective decision-making procedures inspired by the Borda–Kendall method, such as the proposals of Nanson and Copeland (see Fishburn, 1977). For a complete revision of this class of procedures, see, the work of Chebotarev and Shamis (1998), among others.

In real applications, however, it may be desirable to obtain an evaluation of alternatives without externally specifying the weights associated with the votes. Cook and Kress (1990) state that models involving an imposed set of weights fail to provide a fair overall assessment. Each evaluation that implies the use of an externally imposed scoring vector is somewhat arbitrary. Note that a non-winning alternative with the vector considered for the evaluation could be a winner if another weighting vector is applied.

In addition, in group decision problems in which multiple DMs are involved, it is easier to achieve agreement with respect to a set of feasible weights than with respect to a unique vector (as traditional scoring rules require). In this case, reaching a consensus requires determining the minimum information set that contains the preference relations elicited by the DMs, expressed in terms of bounds or inequalities in the weighting vector's components. These two issues help derive the models in which the weights (or scores) associated with each rank position are variables in the problem instead of models that evaluate alternatives with an a priori scoring vector.

Among this class of procedures, I highlight those that are based on Data Envelopment Analysis (DEA). Cook and Kress (1990) propose the initial preference-aggregation model based on a DEA methodology to avoid subjectivity in the choice of a vector of weights. The model evaluates each alternative according to its most favourable scoring vector. Following the philosophy of DEA, each alternative is allowed its own vector of weights to be evaluated. The authors proposes a DEA/AR model, in which the number of votes received at a particular rank position represents an output,

* Tel.: +34 954349355; fax: +34 95434339.
E-mail addresses: iconrub@upo.es, iconrub@gmail.com

0957-4174/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2010.07.002

and the model considers a unique input equal to unity. For the weights, some constraints assure that a better rank position yields a larger weight when included in the model. This set of constraints is denoted in DEA terminology as an assurance region (AR).

With respect to DEA-based models, two main criticisms appear in the literature: multiple top-ties and overly diverse weights. DEA models use assignments of the same aggregate value (equal to unity) to evaluate multiple alternatives as efficient. There is no criterion to discriminate among these alternatives in order to construct a ranking of alternatives. Hence, this approach assumes a drawback to those procedures in which the goal is to construct a rank order of alternatives.

On the other hand, in this procedure, overly diverse weights can appear, given that each alternative can have its own vector of weights (i.e., the one that maximises its aggregate value). The social ranking, which uses different weights for various alternatives, might be rather unacceptable because the traditional ranking uses common weights across alternatives (see Hashimoto & Wu, 2003 for a detailed discussion of this aspect of the approach).

To resolve these two tasks, alternative models have been developed. In Adler, Friedmand, and Sinvany-Stern (2002), the authors summarise the methods proposed for ranking alternatives in the DEA context, yielding a solution to the problem of multiple top-ties and then classifying them into six categories. In the case of a preference-aggregation problem, certain DEA-based procedures have been discussed that, in different ways, attempt to avoid the aforementioned drawback. A complete review of the main models can be seen, for instance, in Llamazares and Pea (2009).

The initial model developed by Cook and Kress (1990) assures that better rank positions yield larger weights by including a discriminating intensity function. In order to construct a ranking of alternatives, the proposed model maximises the discriminating factor. A discrimination procedure based on the cross-evaluation is proposed in Green, Doyle, and Cook (1996) and in Sexton, Silkman, and Hogan (1986) to avoid the problem of choosing a discrimination function that appears in the previous procedure. In Obata and Ishii (2003), the authors demonstrate that in order to compare the aggregate values received by alternatives, it is first necessary to use a weighting vector with the same size. For this reason, they propose normalising the vectors of weights.

In order to discriminate among efficient alternatives, Hashimoto (1997) addresses an AR/exclusion model based on the concept of super-efficiency proposed in Andersen and Petersen (1993). To avoid the problem of diverse weights, the author proposes in Hashimoto and Wu (2003) a model that results in a common vector of weights for evaluating the set of proposed alternatives. More recently, the work of Zerafat Angiz, Emrouznejad, Mustafa, and Rashidi Komijan (2009) introduces a new mathematical method, inspired by DEA methodology, which determines the importance of rank positions according to decision makers in order to reach a more realistic solution.

In this paper, I propose a new model inspired by DEA methodology. The procedure does not specify an a priori vector and consists of two stages. First, a DEA-inspired model for the aggregation of preferences is applied, wherein the objective is not the maximisation of the aggregated value but rather the ordinal position induced by these values. Second, in order to obtain a group solution, the procedure derives a compromise solution by determining a social vector of weights for evaluating the complete set of alternatives.

The rest of the paper is organised as follows. In Section 2, the proposed procedure is described, including the derivation of a linear model to determine the solution in both stages. In Section 3, the model is adapted to a case in which ties between alternatives are permitted, generating weak orders in both stages. Section 4 includes an illustrative example, and Section 5 draws conclusions.

2. Aggregating preferences with a DEA-inspired procedure

Let $X = \{x_1, \ldots, x_N\}$ be a set of N ($N \geqslant 3$) alternatives that have been evaluated by a group of decision makers. Each decisor gives preferences by selecting a subset of K alternatives (or the complete set X) and ranking them from most to least preferred. An order vector can then be derived from the preferences of each DM, such as a vector that contains the names of alternatives in a preference sequence.

From each order vector, a priority or rank vector can be constructed by assigning the position number within the order vector, where the relative position of each alternative is represented in the corresponding order. By convention, the value 1 is assigned to the most important alternative and N to the least important. Note that a priority vector contains the rank values of objects, whereas an order vector contains the names of alternatives. In this paper, rank values will be used for computational purposes, and object names will be used for descriptive purposes.

The aggregation-of-preferences models based on DEA attempt to maximise the aggregate value of votes obtained by each alternative. For each individual alternative, I estimate a maximisation model that gives each alternative the opportunity to have its own scoring vector.

The main idea can be summarised in the following model,

$$\begin{aligned} V^o(x_o) = \max \quad & \sum_{j=1}^{K} w_j^o v_{oj}, \\ \text{s.t.} \quad & \sum_{j=1}^{K} w_j^o v_{ij} \leqslant 1, \quad i = 1, \ldots, N, \\ & w_j^o - w_{j+1}^o \geqslant d(j, \varepsilon), \quad j = 1, \ldots, K-1, \\ & w_K^o \geqslant d(K, \varepsilon). \end{aligned} \tag{1}$$

Model (1) corresponds to the evaluation of alternative x_o. The term w_j^o denotes the weight associated with the jth-rank-position votes in the evaluation of alternative x_o, v_{ij} represents the number of the jth-rank positions votes obtained by alternative x_i, and $V^o(x_i) = \sum_{j=1}^{K} w_j^o v_{ij}$ represents the aggregate value associated with alternative x_i when the optimal weighting vector of alternative x_o is considered. The term $d(j, \varepsilon)$ denotes the discrimination intensity function, where ε represents a non-Archimedean element. This set of constraints implies that the ranking of alternatives has been constructed from most to least preferred, and therefore, the weight assigned to the votes must reflect this feature.

In this context, I propose a two-stage model for determining a group ranking of alternatives. The first stage develops a DEA-inspired model but with a different objective. For each alternative, the model tries to determine a vector of weights that verifies the objective of optimising the rank position of each alternative. That is, the model looks for the weighting vector that minimises the rank position that the alternative occupies in the ranking of alternatives induced by the aggregate values $V^o(x_i)$. The second stage determines a compromise solution for the group after considering the values obtained by each alternative in the previous step, according to each alternative's aspiration levels.

2.1. First stage: optimisation of the ranking position

In the first stage, each alternative is evaluated individually, similar to model (1) but with a different objective. Taking into account the fact that the target of this class of procedures is to construct a ranking of alternatives and the fact that the numeric values of $V^o(x_i)$ are used only for comparative purposes, it is more appropriate to minimise the rank position to describe what the best attainable situation for an alternative is.