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Fuzzy process capability analyses with fuzzy normal distribution

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ABSTRACT

Process capability indices (PCIs) are very a useful statistical analysis tool to summarize process' dispersion and location by process capability analysis (PCA). Additionally PCA produces another two more important results that show us the process ability satisfying specification limits (SLs) and the ratios of conforming (CIs) and nonconforming (NCIs) items which are the probabilities of producing within and out of SLs. However, there are some limitations which prevent a deep and flexible analysis because of the crisp definition of SLs. In this paper, the fuzzy set theory is used to add more sensitiveness to PCA including more information and flexibility. For this aim, fuzzy normal distribution with crisp SLs is first used to calculate the fuzzy percentages of conforming (FCIs) and nonconforming (FNCIs) items by taking into account fuzzy process mean, $\tilde{\mu}$ and fuzzy variance, $\tilde{\sigma}^2$ which are obtained by using fuzzy extension principles. The calculation of the percentages of CIs and NCIs items in fuzzy numbers gains more flexible evaluation ability for the process engineer. Then fuzzy SLs are used together with $\tilde{\mu}$ and $\tilde{\sigma}^2$ to produce fuzzy PCIs (FPCIs) and fuzzy normal distribution. The fuzzy formulation of the indices C_p and C_{pk}, most used two traditional PCIs, are produced when SLs are either triangular (TFN) or trapezoidal fuzzy numbers (TrFN). The proposed methodologies are applied in a piston manufacturer in Konya's Industrial Area, Turkey. FPCIs, FCIs, and FNCIs ratios are determined for piston diameter measurements. The results show that fuzzy estimations of PCIs, CIs, and NCIs have much more treasure to evaluate the process when it is compared with the crisp case.

1. Introduction

Process capability can be broadly defined as the ability of a process to meet customer expectations which are defined as specification limits (SLs). Some of the processes have a success for meeting SLs and therefore are classified as "capable process", while others have not and are classified as "incapable process". The measure of process capability summarizes some aspects of a process's ability to meet SLs. The process capability analysis (PCA) is an approach to define a relationship between the process' ability and SLs.

The PCA compares the output of a process to the SLs by using process capability indices (PCIs). This comparison is made by forming the ratio of the width between the process specification limits to the width of the natural tolerance limits which is measured by six process standard deviation units (Montgomery, 2005). In the literature some PCIs such as $C_p, C_{pk}, C_{pm}, C_{pkm}, C_{pc}, C_{pkc}$ and C_a have been used to measure the ability of process to help us to decide how well the process meets the specification limits. In this paper, the indices C_p and C_{pk} which are well known and most used two main PCIs are analyzed to evaluate the percentages of conforming items (CIs) and nonconforming items (NCIs) and to produce fuzzy PCIs (FPCIs).

C_p which is the first process capability index (PCI) to appear in the literature and called precision index (Kane, 1986) is defined as the ratio of specification width (USL–LSL) over the process spread (6σ). The specification width represents customer and/or product requirements. The process variations are represented by the specification width. If the process variation is very large, the C_p value is small and it represents a low process capability. C_p indicates how well the process fits within the two specification limits. It never considers any process shift as presented in Fig. 1 and it is calculated by using Eq. (1). C_p simply measures the spread of the specifications relative to the six-sigma spread in the process (Kotz & Johnson, 2002; Montgomery, 2005).

$$C_p = \frac{\text{Specification Width}}{\text{Process Spread}} = \frac{\text{Allowable Process Spread}}{\text{Actual Process Spread}} = \frac{\text{USL}-\text{LSL}}{6\sigma} \qquad (1)$$

where σ is the standard deviation of the process. USL and LSL are upper and lower specification limits, respectively. In Fig. 1, C_p indicates how a process confirms to its specifications.

The value of index C_p gives us an opinion about process' performance. For example if it is greater than $1.3\bar{3}$ which corresponds to

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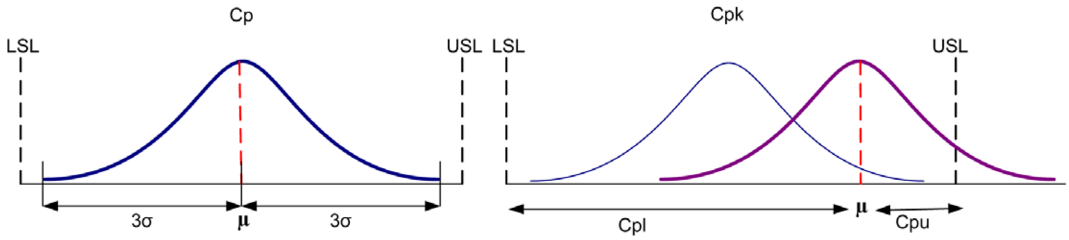

Fig. 1. C_p and C_{pk} indices.

NC = 63 parts per million (ppm) for a centered process, we conclude that process performance is satisfactory. The six quality conditions and the corresponding C_p values are summarized in Table 1 (Tsai & Chen, 2006).

The process capability ratio C_p does not take into account where the process mean is located relative to specifications (Montgomery, 2005). C_p focuses the dispersion of the studied process and does not take into account the centering of the process and thus gives no indication of the actual process performance. Kane (1986) introduced index C_{pk} to overcome this problem. The index C_{pk} is used to provide an indication of the variability associated with a process. It shows how a process confirms to its specification. The index is usually used to relate the "natural tolerances (3σ)" to the specification limits. C_{pk} describes how well the process fits within the specification limits, taking into account the location of the process mean as shown in Fig. 1. C_{pk} should be calculated based on Eqs. (2)–(4) (Kane, 1986; Kotz & Johnson, 2002; Montgomery, 2005).

$$C_{pk} = \min\{C_{pl}, C_{pu}\} \tag{2}$$

$$C_{pl} = \frac{(\mu - \mathrm{LSL})}{3\sigma} \tag{3}$$

$$C_{pu} = \frac{(\mathrm{USL} - \mu)}{3\sigma} \tag{4}$$

The process capability is defined as the percentage of the products which are within in the SLs. It is known that products or process' outputs are inspected with respect to SLs and are classified into two categories: accepted (conforming) and rejected (nonconforming). Consequently the percentages of CIs and NCIs are two basic criteria for interpreting process' ability or performance. The percentage of the CIs can be calculated by using Eq. (5):

Table 1
Quality conditions and C_p values.

Quality condition	C_p value
Super excellent	$2.00 \leqslant C_P$
Excellent	$1.67 \leqslant C_P \leqslant 2.00$
Satisfactory	$1.33 \leqslant C_P \leqslant 1.67$
Capable	$1.00 \leqslant C_P \leqslant 1.33$
Inadequate	$0.67 \leqslant C_P \leqslant 1.00$
Poor	$C_P \prec 0.67$

$$\mathrm{CIs} = \int_{\mathrm{LSL}}^{\mathrm{USL}} P(x)dx = P(\mathrm{USL}) - P(\mathrm{LSL}) \tag{5}$$

where $P(x)$ is the cumulative distribution function of the observed characteristics, X. If the X fits normal distribution, $N(\mu, \sigma^2)$, the percentage of NCIs can be calculated by using Eq. (6):

$$\mathrm{NCIs} = \Phi\left(\frac{\mathrm{LSL} - \mu}{\sigma}\right) + \left[1 - \Phi\left(\frac{\mathrm{USL} - \mu}{\sigma}\right)\right] \tag{6}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution and μ is the process mean. Also the proportion of the CIs can be calculated by $\mathrm{CIs} = 1 - \mathrm{NCIs}$. The NCIs and CIs are shown in Fig. 2.

The percentage of NCIs helps us to produce the sigma quality level of the process and ppm values. Table 2 lists various values of C_p, the corresponding values of standard normal variable Z, and the fractions of nonconformity (defect rate) in ppm for a centered process.

After the inception of the notion of fuzzy sets by Zadeh (1965), many authors have applied this approach to very different areas such as statistics, quality control, and optimization techniques. These studies also affected process capability analyses. In recent years, some papers which have concentrated on different area of PCIs using fuzzy set theory have been published. These areas are as follows: Estimation of the index C_{pk} using Buckley's approach (Wu, 2009), estimation of the indices C_p and C_{pk} when SLs and σ^2 are fuzzy (Kaya & Kahraman, 2008; Kahraman & Kaya, 2009b), analyzing PCIs when the parameters have a correlation (Kaya & Kahraman, 2009b), fuzzy process accuracy index ($\widetilde{C}_a$) (Kahraman & Kaya, 2009a), estimation of FPCIs on six-sigma approach (Kaya & Kahraman, 2009a), multi-process capability analysis using fuzzy inference (Chen & Chen, 2008), fuzzy estimation of the loss-based process capability index (C_{pm}) (Hsu & Shu, 2008), estimation of FPIs using Buckley's approach (Parchami & Mashinchi, 2007), determination of the fuzzy confidence interval (Parchami, Mashinchi, & Maleki, 2006), construction of membership function of C_p and hypothesis testing (Tsai & Chen, 2006), construction of membership functions of PCIs when the SLs are triangular fuzzy numbers (TFN) (Parchami, Mashinchi, Yavari, & Maleki, 2005), estimation of PCIs using fuzzy inference (Chen, Chen, & Lin, 2003), FPCIs and selection of the best supplier (Chen, Lin, & Chen, 2003), calculation of the FPCIs when observations are

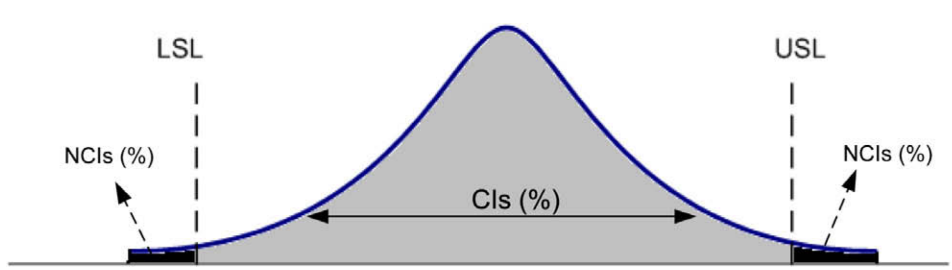

Fig. 2. Conforming and nonconforming items.