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Finite-time quantized guaranteed cost fuzzy control for continuous-time nonlinear systems

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ABSTRACT

This paper considers the problem of finite-time quantized guaranteed cost fuzzy control for continuous-time nonlinear systems. Firstly, the definition on finite-time stability (FTS) for continuous-time nonlinear systems is provided and we give a novel and explicit interpretation for finite-time quantized guaranteed cost control. Secondly, sufficient conditions for the existence of state feedback controller are derived in terms of linear matrix inequities (LMIs), which guarantee the requirements of the provided performance criterion. The related optimization problem is also offered to minimize the guaranteed cost performance bound. Finally, an illustrative example is presented to show the validity of the proposed scheme.

1. Introduction

Since there exist many cases where the state values are restrained, we generally need to ensure that these state values are allowable. In order to deal with these cases, the concept of finite-time stability (FTS) (or short-time stability) can be introduced into our eyeshot. As illuminated in Amato, Ariola, and Dorato (2001), a system is said to be finite-time stable if, once we fix a time-interval, its state does not exceed certain bound during this time-interval. Some early results on FTS can be found in Dorato (1961), Weiss and Infante (1967). More recently the problem of finite-time control for both continuous-time and discrete-time linear systems has been investigated in Amato and Ariola (2005), Amato, Ariola, and Cosentino (2006), Amato et al. (2001) via linear matrix inequities (LMIs) technique. In the existing results, FTS is mainly associated with linear systems. How to solve the FTS problem for nonlinear systems is still a difficult problem.

Recently, Takagi–Sugeno (T–S) fuzzy model proposed in Takagi and Sugeno (1985) is widely applied in various industrial control fields because of its simple structure with local dynamics. The typical approach named as parallel distributed compensation (PDC) (Wang, Tanaka, & Griffin, 1996) is also developed to design the fuzzy controllers. Moreover, besides the stabilization problem, the guaranteed cost fuzzy control was extendedly investigated to stabilize the controlled systems while providing an upper bound on a given performance index by many scholars in the last decade, such as Chen and Liu (2005), Chen, Liu, Tong, and Lin (2007), Guan and Chen (2004), Jiang and Han (2007).

The output signals of the controller are generally assumed to be returned directly to the plant without data loss in the classical feedback control theories. In practice, however, this is not true in many real applications since all data transmissions cannot be performed with infinite precision in computer control systems and quantization always exists. In recent years, considering the limited communication capacity, the problem of quantized feedback control has been investigated by many researchers, such as Brockett and Liberzon (2000), Elia and Mitter (2001), Fu and Xie (2005), Liberzon (2003). Moreover, how to design a quantized feedback controller for nonlinear systems is still an interesting issue.

In this paper, we propose a finite-time quantized guaranteed cost fuzzy control scheme for continuous-time nonlinear systems. Its objective is to find a suitable quantized controller such that the provided performance criterion is satisfied. As far as we know, up to date, the finite-time quantized guaranteed cost control for continuous-time nonlinear systems has not been considered in the existing results.

The remainder of this paper is organized as follows: Basic problem formulation is introduced in Section 2. The finite-time quantized guaranteed cost controller via state feedback is designed in Section 3. Section 4 provides an illustrative example to demonstrate the effectiveness of the proposed scheme. Finally, concluding remarks are made in Section 5.

In the following sections, the identity matrices and zero matrices are denoted by I and 0, respectively. X^T denotes the transpose of matrix X. R^n denotes the n-dimensional Euclidean space. R_+ denotes the positive real number. The notation * always denotes the symmetric block in one symmetric matrix. The standard notation > (<) is used to denote the positive (negative)-definite ordering of matrices. Inequality $X > Y$ shows that the matrix $X - Y$ is positive

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definite. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ indicate the maximum and minimum eigenvalues of the matrix, respectively.

2. Problem formulation

Consider a general class of continuous-time nonlinear systems which can be represented by the following T–S fuzzy dynamic model.

Plant Rule i
IF $\theta_1(t)$ is $N_{i1},\ldots,$ and $\theta_p(t)$ is N_{ip}
THEN
$\dot{x}(t) = A_i x(t) + B_i u(t)$

where $i = 1,2,\ldots,r$ is the number of fuzzy rules, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are known constant matrices, of the ith subsystem, $\theta_1(t),\theta_2(t),\ldots,\theta_p(t)$ are the premise variables, N_{ij} is the fuzzy set $(j = 1,2,\ldots,p)$. By using singleton fuzzifier, product inference and center-average defuzzifier, the fuzzy dynamic model is expressed by

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + B_i u(t)], \tag{1}$$

where $h_i(\theta(t)) = \mu_i(\theta(t))/\sum_{i=1}^{r}\mu_i(\theta(t)),\ \mu_i(\theta(t)) = \Pi_{j=1}^{p}N_{ij}(\theta_j(t))$ and $N_{ij}(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in N_{ij}. It is assumed that $\mu_i(\theta(t)) \geqslant 0$, and $\sum_{i=1}^{r}\mu_i(\theta(t)) > 0$ for all t. Then we can see that $h_i(\theta(t)) \geqslant 0$ and $\sum_{i=1}^{r}h_i(\theta(t)) = 1$.

According to the conventional PDC concept, we design the following fuzzy controller via state feedback:

Controller Rule i
IF $\theta_1(t)$ is $N_{i1},\ldots,$ and $\theta_p(t)$ is N_{ip}
THEN
$u(t) = K_i x(t)$

where $i = 1,2,\ldots,r$ is the number of controller rules, $K_i \in R^{m\times n}$ is the controller gain matrix. Thus, the fuzzy controller can be expressed as follows:

$$u(t) = \sum_{i=1}^{r} h_i(\theta(t))K_i x(t). \tag{2}$$

It is assumed that the output signals of fuzzy controller (2) are passed via a quantizer and the quantizer is denoted as $f(\cdot) = [f_1(\cdot),f_2(\cdot),\ldots,f_m(\cdot)]^T$, which is assumed to be symmetric, that is, $f(-v) = -f(v)$. The set of quantized levels is described by

$$\Omega = \{\pm u_i,\ i = 0,\pm 1,\pm 2,\ldots\} \cup \{0\}.$$

According to Elia and Mitter (2001) and Fu and Xie (2005), a quantizer is called logarithmic if the set of quantized levels is characterized by

$$\Omega = \{\pm u_i, u_i = \rho_i u_0,\ i = \pm 1,\pm 2,\ldots\} \cup \{\pm u_0\} \cup \{0\},$$
$$0 < \rho_i < 1, \quad u_0 > 0.$$

For the logarithmic quantizer, the associated quantizer $f(\cdot)$ is defined as follows:

$$f(v) = \begin{cases} u_i, & \text{if } \frac{1}{1+\sigma}u_i < v \leqslant \frac{1}{1-\sigma}u_i, \quad v > 0, \\ 0, & \text{if } v = 0, \\ -f(-v), & \text{if } v < 0. \end{cases}$$

where

$$\sigma = \frac{1-\rho}{1+\rho}.$$

Considering the above quantization behavior, we can obtain the following quantized fuzzy controller:

$$\bar{u}(t) = f\left(\sum_{i=1}^{r} h_i(\theta(t))K_i x(t)\right). \tag{3}$$

Moreover, (3) can be expressed as

$$\bar{u}(t) = (I + \Lambda)\sum_{i=1}^{r} h_i(\theta(t))K_i x(t), \tag{4}$$

where $\Lambda = \mathrm{diag}\{\Lambda_1,\Lambda_2,\ldots,\Lambda_m\}$ and $\Lambda_s \in [-\sigma,\sigma],\ s = 1,\ldots,m$. The closed-loop system (1) can be reconstructed under the quantized fuzzy controller (4) as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + B_i\bar{u}(t)] = \overline{A}x(t), \tag{5}$$

where $\overline{A} = \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t))[A_i + B_i(I+\Lambda)K_j]$.

Next we will provide a novel concept on FTS for continuous-time nonlinear systems. It is formalized through the following definition. Similar description can be found in Amato et al. (2001).

Definition 1. The closed-loop continuous-time nonlinear system (5) is said to be finite-time stable (FTS) with respect to (c_1,c_2,T,R_C) where $0 < c_1 < c_2, T \in R_+$ and $R_C > 0$ if

$$x^T(0)R_C x(0) \leqslant c_1 \Rightarrow x^T(t)R_C x(t) < c_2, \quad \forall\ t \in (0,T].$$

Remark 1. Asymptotic stability and FTS are independent concepts each other as it was described in Amato et al. (2001). In certain cases, asymptotic stability can be also considered as an additional requirement while restricting our attention on a finite-time interval.

Furthermore, for (5), we provide the following cost function associated with the above definition:

$$J = \int_0^T \left[x^T(t)Q_1 x(t) + \bar{u}^T(t)Q_2\bar{u}(t)\right]dt, \tag{6}$$

where $Q_1 > 0$ and $Q_2 > 0$ are given weighting matrices or given positive scalars. The following novel and explicit interpretation for finite-time quantized guaranteed cost control is given by:

Definition 2. For (5), if there exists a quantized fuzzy control law (4) and a scalar ψ_0 such that the closed-loop system is finite-time stable and the value of the cost function (6) satisfies $J < \psi_0$, then ψ_0 is said to be a guaranteed cost bound and the designed control law (4) is said to be a finite-time quantized guaranteed cost fuzzy control law.

3. Main results

In this section, the problem of finite-time quantized guaranteed cost fuzzy control via state feedback is studied according to (5). Some results are provided as follows.

Theorem 1. *The closed-loop continuous-time nonlinear system (5) is finite-time stable with respect to* (c_1,c_2,T,R_C) *where* $0 < c_1 < c_2, T \in R_+, R_C > 0$, *and has the guaranteed cost bound W if there exist a scalar* $\alpha \geqslant 0$, *matrix* $P > 0 \in R^{n\times n}$ *and the controller gain matrix* $K_j \in R^{m\times n}$ *such that the following conditions are satisfied*

$$\begin{aligned} &\Psi_{ii} < 0, \quad 1 \leqslant i \leqslant r, \\ &\Omega_{ij} < 0, \quad 1 \leqslant i < j \leqslant r, \end{aligned} \tag{7}$$

$$\frac{c_1}{\lambda_{\min}(P)} \leqslant \frac{c_2}{\lambda_{\max}(P)e^{\alpha T}}, \tag{8}$$