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Hesitant triangular fuzzy information aggregation based on Einstein operations and their application to multiple attribute decision making

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ABSTRACT

In this paper, we investigate the multiple attribute decision making (MADM) problems in which attribute values take the form of hesitant triangular fuzzy information. Firstly, definition and some operational laws of hesitant triangular fuzzy elements and score function of hesitant triangular fuzzy elements are introduced. Then, we have developed some hesitant triangular fuzzy aggregation operators based on the Einstein operation: the hesitant triangular fuzzy Einstein weighted averaging (HTFEWA) operator, hesitant triangular fuzzy Einstein weighted geometric (HTFEWG) operator, hesitant triangular fuzzy Einstein ordered weighted averaging (HTFEOWA) operator, hesitant triangular fuzzy Einstein ordered weighted geometric (HTFEOWG) operator, hesitant triangular fuzzy Einstein hybrid average (HTFEHA) operator and hesitant triangular fuzzy Einstein hybrid geometric (HTFEHG) operator. We have applied the hesitant triangular fuzzy Einstein weighted averaging (HTFEWA) operator, hesitant triangular fuzzy Einstein weighted geometric (HTFEWG) operators to multiple attribute decision making with hesitant triangular fuzzy information. Finally an illustrative example has been given to show the developed method.

1. Introduction

The theory of fuzzy sets (FSs) proposed by Zadeh (1965) has achieved a great success in various fields. Out of several higher order FSs, intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1986) have been found to be highly useful to deal with vagueness. The concept of IFSs is a generalization of that of FSs. The concept of vague sets (VSs) introduced by Gau and Buehrer (1993) is another generalization of fuzzy sets. But, Bustince and Burillo (1996) point out that the notion of VSs is the same as that of IFSs. Another, well-known generalization of an ordinary fuzzy set is so-called interval-valued fuzzy sets (IVFSs). Generally, the idea of IVFSs was attributed to Gorzaleczany (1987) and Turksen (1986), but actually there is a strong connection between IFSs and IVFSs. Atanassov and Gargov (1989) introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) as a further generalization of that of IFSs, as well as of IVFSs. Atanassov (1994) defined some operational laws of IVIFSs. The intuitionistic fuzzy set and interval-valued intuitionistic fuzzy sets has received more and more attention since its appearance (Chen, 2010; Chen and Li, 2010; Chen et al., 2011; Li, 2010a, 2010b, 2011; Wei, 2008, 2009, 2010a, 2010b; Xu, 2007, 2010; Xu and Wang, 2012; Xu and Yager, 2006, 2011; Zeng and Su, 2011). Furthermore, Torra (2010) proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu (2011) defined some hesitant fuzzy operational rules and proposed a series of operators under various situations and discussed the relationships among them. Xu et al. (2011) developed several series of aggregation operators for hesitant fuzzy information with the aid of quasi-arithmetic means. Wei et al. (2012) developed two hesitant fuzzy Choquet integral aggregation operators: hesitant fuzzy Choquet ordered averaging (HFCOA) operator and hesitant fuzzy Choquet ordered geometric (HFCOG) operator and applied the HFCOA and HFCOG operators to multiple attribute decision making with hesitant fuzzy information. Furthermore, they propose the generalized hesitant fuzzy Choquet ordered averaging (GHFCOA) operator and generalized hesitant fuzzy Choquet ordered geometric (GHFCOG) operator. Motivated by the ideal of prioritized aggregation operators (Yager, 2008, 2009), Wei (2012) develop some prioritized aggregation operators for aggregating hesitant fuzzy information, and then apply them to develop some models for hesitant fuzzy multiple attribute decision making (MADM) problems in which the attributes are in different priority level. Zhu et al. (2012) define the hesitant fuzzy geometric Bonferroni mean (HFGBM), the hesitant fuzzy Choquet geometric Bonferroni mean (HFCGBM), the weighted hesitant fuzzy geometric Bonferroni mean (WHFGBM) and the weighted hesitant fuzzy Choquet geometric Bonferroni mean (WHFCGBM) and apply the proposed aggregation operators to multi-criteria decision making, and give some examples to illustrate their results. Gu et al. (2011) investigated the evaluation model for risk investment with hesitant fuzzy information. They utilized the hesitant fuzzy weighted averaging (HFWA) operator to aggregate

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the hesitant fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function. Xu and Xia (2011) proposed a variety of distance measures for hesitant fuzzy sets and developed a number of hesitant ordered weighted distance measures and hesitant ordered weighted similarity measures which can alleviate the influence of unduly large (or small) deviations on the aggregation results by assigning them low (or high) weights. Xu and Xia (2011) defined the distance and correlation measures for hesitant fuzzy information and then discuss their properties in detail. Wang et al. (2011) we propose the generalized hesitant fuzzy hybrid weighted distance (GHFHWD) measure, which is based on the generalized hesitant fuzzy weighted distance (GHFWD) measure and the generalized hesitant fuzzy ordered weighted distance (GHFOWD) measure proposed by Xu and Xia (2011).

From above analysis, we can see that hesitant fuzzy set is a very useful tool to deal with uncertainty. More and more multiple attribute group decision making theories and methods under hesitant fuzzy environment have been developed. Current methods are under the assumption that hesitant fuzzy set permits the membership having a set of possible exact and crisp values. However, under many conditions, for the real multiple attribute group decision making problems, the decision information about alternatives is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human think, thus, exact and crisp values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated which permits the membership having a set of possible triangular fuzzy values. So, in this paper we shall propose the concept of the hesitant triangular fuzzy set based on hesitant fuzzy set to overcome this limitation. To do so, the remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to hesitant fuzzy sets and some operational laws of hesitant fuzzy elements. In Section 3 we define some basic concepts related to hesitant triangular fuzzy sets and some operational laws of hesitant triangular fuzzy elements. In Section 4 we shall develop some aggregating operators with hesitant triangular fuzzy information. In Section 5, furthermore, we shall develop some aggregating operators based on the Einstein operations with hesitant triangular fuzzy information. In Section 6, we shall apply the hesitant triangular fuzzy Einstein weighted averaging (HTFEWA) operator and hesitant triangular fuzzy Einstein weighted geometric (HTFEWG) operator to multiple attribute decision making with hesitant triangular fuzzy information. In Section 7, an illustrative example is pointed out. In Section 8, we conclude the paper and give some remarks.

2. Preliminaries

Definition 1. Let X be an universe of discourse, then a fuzzy set is defined as:

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\} \tag{1}$$

which is characterized by a membership function $\mu_A{:}X \to [0,1]$, where $\mu_A(x)$ denotes the degree of membership of the element x to the set A (Zadeh, 1965).

Atanassov extended the fuzzy set to the IFS, shown as follows:

Definition 2. An IFS A in X is given by

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\} \tag{2}$$

where $\mu_A{:}X \to [0,1]$ and $v_A{:}X \to [0,1]$, with the condition $0 \leqslant \mu_A(x) + v_A(x) \leqslant 1$, $\forall\ x \in X$. The numbers $\mu_A(x)$ and $v_A(x)$ represents, respectively, the membership degree and non-membership degree of the element x to the set A (Atanassov, 1986).

Definition 3. For each IFS A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x), \quad \forall x \in X. \tag{3}$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A (Atanassov, 1986).

However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra (2010) proposed another generation of FS.

Definition 4 Torra (2010). Given a fixed set X, then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a sunset of $[0,1]$.

To be easily understood, Xia and Xu (2011) express the HFS by mathematical symbol:

$$E = (\langle x, h_E(x)\rangle | x \in X), \tag{4}$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xia and Xu (2011) call $h = h_E(x)$ a hesitant fuzzy element (HFE) and H the set of all HFEs.

Definition 5 Xia and Xu (2011). For a HFE h, $s(h) = \frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu (2011) define some new operations on the HFEs h, h_1 and h_2:

(1) $h^\lambda = \cup_{\gamma\in h}\{\gamma^\lambda\}$;
(2) $\lambda h = \cup_{\gamma\in h}\{1-(1-\gamma)^\lambda\}$;
(3) $h_1 \oplus h_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1+\gamma_2-\gamma_1\gamma_2\}$;
(4) $h_1 \otimes h_2 = \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\{\gamma_1\gamma_2\}$.

3. Hesitant triangular fuzzy set (HTFS)

3.1. Triangular fuzzy numbers

In this section, we briefly describe some basic concepts and basic operational laws related to triangular fuzzy numbers.

Definition 6 Van Laarhoven and Pedrycz (1983). A triangular fuzzy numbers $\tilde{a}$ can be defined by a triplet (a^L, a^M, a^U).The membership function $\mu_{\tilde{a}}(x)$ is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a^L, \\ \frac{x-a^L}{a^M-a^L}, & a^L \leqslant x \leqslant a^M, \\ \frac{x-a^U}{a^M-a^U}, & a^M \leqslant x \leqslant a^U, \\ 0, & x \geqslant a^U. \end{cases} \tag{5}$$

where $0 < a^L \leqslant a^M \leqslant a^U$, a^L and a^U stand for the lower and upper values of the support of $\tilde{a}$,respectively, and a^M for the modal value.

Definition 7 Van Laarhoven and Pedrycz (1983). Basic operational laws related to triangular fuzzy numbers:

$$\tilde{a} \oplus \tilde{b} = \left[a^L, a^M, a^U\right] \oplus \left[b^L, b^M, b^U\right] = \left[a^L + b^L, a^M + b^M, a^U + b^U\right]$$

$$\tilde{a} \otimes \tilde{b} = \left[a^L, a^M, a^U\right] \otimes \left[b^L, b^M, b^U\right] = \left[a^L b^L, a^M b^M, a^U b^U\right]$$

$$\lambda \otimes \tilde{a} = \lambda \otimes \left[a^L, a^M, a^U\right] = \left[\lambda a^L, \lambda a^M, \lambda a^U\right], \quad \lambda > 0.$$