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An effective hybrid quantum-inspired evolutionary algorithm for parameter estimation of chaotic systems

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ABSTRACT

Parameter estimation of chaotic systems is an important issue and has attracted increasing interest from a variety of research fields. Recently, quantum-inspired evolutionary algorithms have been proposed and applied to some optimization problems. However, to the best of our knowledge, there is no published research work on quantum-inspired evolutionary algorithm (QEA) for estimating parameters of chaotic systems. In this paper, an effective hybrid quantum-inspired evolutionary algorithm with differential evolution (HQEDE) is proposed and applied to estimate the parameters of the Lorenz system. Numerical simulation and comparisons with other methods demonstrate the effectiveness and robustness of the proposed algorithm. In addition, the effects of the parameter settings on HQEDE are investigated.

1. Introduction

As a quintessence of nonlinear systems, chaos is characterized by an unstable dynamic behavior that exhibits sensitive dependence on the initial condition and includes infinite unstable periodic motions. Potential applications of chaos have been showed in many academic and engineering fields, such as communications, economic system and optimization (Chen & Dong, 1998). Meanwhile, control and synchronization of chaotic systems have been investigated in a variety of fields during the past two decades (Hubler, 1989; Ott, Grebogi, & Yorke, 1990). However, many existing approaches depend on some parameters that may be difficult to determine in advance due to the complexity of chaotic systems. Therefore, parameter estimation of chaotic systems has become an important issue of nonlinear science. So far, many methods have been proposed for parameter estimation of chaotic systems (Dai, Ma, Li, & You, 2002; He, Wang, & Liu, 2007; Inés & Joaquín, 2006; Li, Yang, Peng, & Wang, 2006; Peng, Liu, Zhang, & Wang, 2009; Rahul, 2005).

Recently, evolution algorithms have been applied to parameter estimation of chaotic systems by formulating the problem as a multi-dimensional optimization problem. Genetic algorithm (GA) was adopted to estimate parameters of the Lorenz system (Dai et al., 2002), and chaotic ant swarm algorithm was applied to the Logistic iteration system and the Lorenz system (Li et al., 2006), and particle swarm optimization (PSO) algorithm (He et al., 2007) and differential evolution (DE) (Peng et al., 2009) were proposed for parameter estimation of the Lorenz system.

Based on the principle of quantum mechanics and computing science, quantum computing has been a novel computing technique with some superiority to classical methods. By fusing the quantum computing and evolutionary algorithm, quantum-inspired evolutionary algorithm (QEA) was proposed (Han & Kim, 2002) and has been an effective optimization technique for many complex optimization problems, such as numerical optimization problems and knapsack problems (Han & Kim, 2002, 2004), parameter estimation of nonlinear systems (Wang, Tang, & Wu, 2005), and scheduling problems (Li & Wang, 2007). It has been shown by numerical simulation results that QEAs are of better performances than classical EAs in terms of convergence rate, searching quality, and robustness. To the best of our knowledge, there is no research work about QEA for parameter estimation of chaotic systems. By hybridizing the QEA and differential evolution, an effective hybrid quantum-inspired algorithm is proposed in this paper for parameter estimation of the chaotic system. Numerical simulations based on the Lorenz system and comparisons with other methods demonstrate the effectiveness, efficiency and robustness of the proposed algorithm.

The remainder of this paper is organized as follows. In Section 2, parameter estimation of the chaotic system is formulated from the optimization point of view. In Section 3, the hybrid algorithm of QEA and DE is proposed. Numerical simulations and comparisons are provided in Section 4. Finally, we end the paper with some conclusions in Section 5.

2. Problem formulation

Considering the following n-dimensional chaotic system:

$$\dot{X} = G(X, X_0, \theta_0) \tag{1}$$

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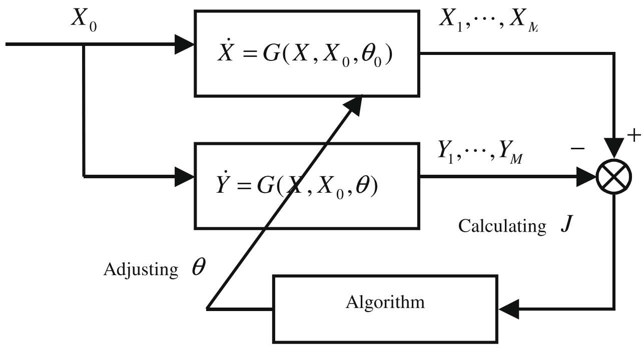

Fig. 1. The principle of parameter estimation for a chaotic system.

where $X = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes an n-dimensional state vector, X_0 denotes an initial state and $\theta_0 = [\theta_{10}, \theta_{20}, \ldots, \theta_{n0}]^T$ is a set of original parameters.

Suppose that the structure of the system is known in advance. Thus, the estimated system can be described as follows:

$$\dot{Y} = G(X, X_0, \theta) \tag{2}$$

where $Y = [y_1, y_2, \ldots, y_n]^T \in R^n$ denotes a state vector and $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T$ is a set of estimated parameters.

The parameter estimation problem can be formulated as the following optimization problem:

$$\min \ J = \frac{1}{M} \sum_{k=1}^{M} \|X_k - Y_k\| \text{ by searching suitable } \theta \tag{3}$$

where M denotes the length of data used for parameter estimation, X_k and $Y_k (k = 1, 2, \ldots, M)$ denote the states of the original system and the estimated system at time k respectively.

Obviously, the parameter estimation of a chaotic system is a multi-dimensional continuous optimization problem, where the decision vector is θ and the optimization goal is to minimize J. The principle of parameter estimation for a chaotic system from the optimization view of point can be illustrated with Fig. 1. The original parameters of a chaotic system are not easy to estimate because of the unstable dynamic of the chaotic systems. Meanwhile, it is very difficult for traditional optimization methods to obtain the original parameters to achieve global optimization, since there are lots of local optima in the landscape of the goal function. Thus, we aim to solve the problem by proposing an effective quantum-inspired evolutionary algorithm in this paper.

3. Hybrid algorithm HQEDE

3.1. Quantum-inspired evolutionary algorithm

Quantum-inspired evolutionary algorithm is based on the concept and principles of quantum mechanics, such as wave function, linear superposition and interference, by employing qubit as the smallest unit of information. The state of a qubit may be in '0' or in '1', or in any superposition of the two, which can be represented as follows:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{4}$$

where α and β are complex numbers that specify the probability amplitudes of the corresponding state. $|\alpha|^2$ and $|\beta|^2$ give the probabilities that the qubit will be in the state '0' and '1', respectively. Normalization to the unity guarantees $|\alpha|^2 + |\beta|^2 = 1$.

Different from the classical EA, QEA uses a qubit-based representation as chromosome. For example, an m-qubit representation is defined as follows:

$$\left| \begin{matrix} \alpha_1 \\ \beta_2 \end{matrix} \middle| \begin{matrix} \alpha_2 \\ \beta_2 \end{matrix} \middle| \begin{matrix} \ldots \\ \ldots \end{matrix} \middle| \begin{matrix} \alpha_m \\ \beta_m \end{matrix} \right| \tag{5}$$

where $|\alpha_i|^2 + |\beta_i|^2 = 1, i = 1, 2, \ldots, m$. This single chromosome contains information of 2^m states and collapses to a single state in the act of observation. With such a representation, it can represent superposition of states.

QEA is a probabilistic algorithm similar to other EAs. However, QEA maintains a population of Q-bit individuals $Q(t) = \{q_1^t, q_2^t, \ldots, q_N^t\}$ at generation t, where N is the population size, and q_j^t is an m-qubit individual defined as follows:

$$q_j^t = \left| \begin{matrix} \alpha_{j1}^t \\ \beta_{j1}^t \end{matrix} \middle| \begin{matrix} \alpha_{jm}^t \\ \beta_{j2}^t \end{matrix} \middle| \begin{matrix} \ldots \\ \ldots \end{matrix} \middle| \begin{matrix} \alpha_{jm}^t \\ \beta_{jm}^t \end{matrix} \right| \tag{6}$$

where m is the number of bits used to code the solution.

The procedure of QEA is described as follows:

Step 1: Let $t = 0$ and set all α_i^0 and β_j^0 of every qubit-string q_j^0 of $Q(t)$ as $1/\sqrt{2}$ or $-1/\sqrt{2}$. In this way, any q_j^0 can represent the linear superposition of all the possible states with the same probability. In addition, with different signs it helps the individuals with good diversity in the process of evolution.

Step 2: By observing $Q(t)$, every q_j^t of $Q(t)$ is transformed to a binary string X_j^t based on the probability amplitudes to form $R(t) = \{r_1^t, r_2^t, \ldots, r_N^t\}$.

Step 3: Transform each individual of $R(t)$ to a solution with real values and use the goal function to get the fitness value, and then store its best solution into $B(t)$.

Step 4: If a terminal condition is satisfied, then output the best solution; else let $t = t + 1$ and continue the following steps.

Step 5: By observing $Q(t-1)$, obtain and evaluate $R(t)$.

Step 6: Update each qubit of all individuals of $Q(t)$ using quantum rotation gates $G(t)$ as follows.

$$\begin{bmatrix} \alpha_i' \\ \beta_i' \end{bmatrix} = G(t) \cdot \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \tag{7}$$

where θ is a rotation angle. For ith qubit of q, i.e. $(\alpha_i, \beta_i), \theta_i = s(\alpha_i, \beta_i)\Delta\theta_i$, where $s(\alpha_i, \beta_i)$ is the sign of θ_i for determining the direction and $\Delta\theta_i$ is the magnitude of rotation angle as shown in Table 1.

Step 7: Store the best solution of $R(t)$ into $B(t)$ and go back to Step 4.

Table 1
Lookup table of rotation angle.

r_i	b_i	$f(r) < f(b)$	$\Delta\theta_i$	$s(\alpha_i, \beta_i)$			
				$\alpha_i\beta_i > 0$	$\alpha_i\beta_i < 0$	$\alpha_i = 0$	$\beta_i = 0$
0	0	False	0	0	0	0	0
0	0	True	0	0	0	0	0
0	1	False	0	0	0	0	0
0	1	True	0.05π	−1	+1	±1	0
1	0	False	0.01π	−1	+1	±1	0
1	0	True	0.025π	+1	−1	0	±1
1	1	False	0.005π	+1	−1	0	±1
1	1	True	0.025π	+1	−1	0	±1

Remark: b_i and r_i are the ith bits of the best solution b and the binary solution r, respectively.