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Economic design of autoregressive moving average control chart using genetic algorithms

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ABSTRACT

When designing control charts, it is usually assumed that the observations from the process at different time points are independent. However, this assumption may not be true for some production processes, e.g., the continuous chemical processes. The presence of autocorrelation in the process data can result in significant effect on the statistical performance of control charts. Jiang, Tsui, and Woodall (2000) developed a control chart, called the autoregressive moving average (ARMA) control chart, which has been shown suitable for monitoring a series of autocorrelated data. In the present paper, we develop the economic design of ARMA control chart to determine the optimal values of the test and chart parameters of the chart such that the expected total cost per hour is minimized. An illustrative example is provided and the genetic algorithm is applied to obtain the optimal solution of the economic design. A sensitivity analysis shows that the expected total cost associated with the control chart operation is positively affected by the occurrence frequency of the assignable cause, the time required to discover the assignable cause or to correct the process, and the quality cost per hour while producing in control or out of control, and is negatively influenced by the shift magnitude in process mean.

1. Introduction

Statistical process control is an effective approach for improving product quality and saving production costs for a firm. Since 1924 when Dr. Shewhart presented the first control chart, various control chart techniques have been developed and widely applied as a primary tool in statistical process control. The major function of control charting is to detect the occurrence of assignable causes so that the necessary corrective action can be taken before a large quantity of nonconforming product is manufactured. The control chart technique may be considered as the graphical expression and operation of statistical hypothesis test. When a control chart is used to monitor a process, some test parameters should be determined, i.e., the sample size, the sampling interval between successive samples, and the control limits or critical region of the chart. Duncan (1956) first proposed a cost function for economically determining the test parameters for the average control chart that minimizes the average cost when a single out-of-control state (assignable cause) exists, which is called the "economic design" of control charts. Duncan's cost function includes the cost of sampling and inspection, the cost of defective products, the cost of false alarm, the cost of searching assignable cause, and the cost of process correction. Since then, considerable attention has been devoted to the economically optimal determination of the test parameters of control charts, e.g., see Montgomery (1980), Vance (1983), Ho and Case (1994a) and Chou, Chen, and Liu (2001). Lorenzen and Vance (1986) also introduced a unified approach for economic design of control charts. Application of Lorenzen and Vance's approach may be found in Torng, Montgomery, and Cochran (1994), Ho and Case (1994b) and Chou, Chen, and Liu (2006).

Traditionally, when designing control charts, it is usually assumed that the observations from the process at different time points are independent. However, this assumption may not be tenable in some production processes, e.g., the continuous chemical processes. The presence of autocorrelation in the process data can result in significant effect on the statistical performance of control charts. The major problem is that variations due to the autocorrelation may produce false out-of-control signals. Excessive false alarms may lead to unnecessary process adjustment and loss of confidence in the control chart as a monitoring tool. Jiang et al. (2000) developed a control chart, called the autoregressive moving average (ARMA) control chart, which has been shown suitable for monitoring a series of autocorrelated data.

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Various solution procedures for economically determining the optimal values of the test parameters of control charts have been developed and applied in the literature. The genetic algorithm (GA), based on the concept of natural genetics, uses the stochastic way (not deterministic rule) to guide the search direction of finding the optimal solution and is able to search for many possible solutions at the same time. Therefore, GA is considered as an appropriate way for solving the problems of combinatorial optimization and has been successfully applied in solution procedure of economic designs of control charts, e.g., see Chou, Wu, and Chen (2006), Chou, Cheng, and Lai (2008) and Lin, Chou, and Lai (2009).

The present paper presents the economic design of ARMA control chart, in which the test and chart parameters are determined such that the average total cost associated with control chart operation is minimized. In the next section, a brief description of the use of ARMA control chart to maintain current control of an autocorrelated process is given. The cost function is then established by applying the cost function in Lorenzen and Vance (1986). The GA is employed to obtain the optimal values of the test and chart parameters for ARMA control chart, and an example is provided to illustrate the solution procedure. A sensitivity analysis is then carried out to investigate the effects of model parameters on the solution of the economic design.

2. The ARMA control chart

The ARMA control chart was developed by Jiang et al. (2000) and has been shown to be effective for monitoring a process with autocorrelated measurements. Suppose that the variable x_t is the measurement at time t from a normal distribution with mean μ and variance σ^2. According to Jiang et al. (2000), for an ARMA process, the measurement x_t at time t can be expressed as a linear combination of the measurement at time $t-1$, the vibration factors at time t (denoted by a_t) and the vibration factors at time $t-1$ (denoted by a_{t-1}), i.e., mathematically:

$$X_t = a_t - v a_{t-1} + u x_{t-1}, \quad \text{for } |u| < 1 \quad \text{and} \quad |v| < 1, \tag{1}$$

where a_i's at time i are normally and independently distributed with mean 0 and variance σ_a^2, the constant u is the autoregressive parameter of the process, and the constant v is the moving average parameter of the process. It can be shown that:

$$\sigma^2 = \frac{1 - 2uv + v^2}{1 - u^2}\sigma_a^2. \tag{2}$$

The sample statistic used in the operation of an ARMA control chart at time t is defined as:

$$Z_t = \theta_0 x_t - \theta x_{t-1} + \phi Z_{t-1} = \theta_0(x_t - \beta x_{t-1}) + \phi Z_{t-1}, \tag{3}$$

where Z_0 is generally the target of the characteristic, θ and ϕ are respectively the moving average parameter and the autoregressive parameter of the ARMA control chart, $\theta_0 = 1 + \theta - \phi$ and $\beta = \theta/\theta_0$. To guarantee that the monitoring process is reversible and stationary, we have the constraints that $|\beta| < 1$ and $|\phi| < 1$. It may be shown that the sample statistic in Eq. (3) has the mean μ and a steady-state variance σ_Z^2, where:

$$\sigma_Z^2 = \left[\frac{2(\theta - \phi)(1 + \theta)}{1 + \theta}\right]\sigma^2. \tag{4}$$

Thus, the upper and lower control limits, abbreviated by UCL and LCL respectively, and the center line (CL) of the ARMA control chart can be calculated by

$$\mathrm{UCL} = \mu + k\sigma_Z, \tag{5}$$

$$\mathrm{CL} = \mu,$$

$$\mathrm{LCL} = \mu - k\sigma_Z. \tag{6}$$

where k is the control limit coefficient and is one of the test parameters in the ARMA control chart to be determined in the economic design. The chart parameters θ and ϕ play important roles in the detection performance for an ARMA chart. In the present paper, the values of θ and ϕ will be also determined based on economic consideration.

3. The cost function

To simplify the mathematical manipulation of the cost function and its corresponding economic design, the following assumptions are made:

(1) The measurements monitored by the ARMA control chart follow the first-order autoregressive and moving average process. That is, the design of ARMA control chart in the present paper is focused on the process of ARMA(1, 1).
(2) In the start of the process, the process is assumed to be in the safe state; that is, $\mu = \mu_0$.
(3) The process mean may be shifted to the out-of-control region due to an assignable cause; that is, $\mu = \mu_0 + \delta\sigma$.
(4) The process standard deviation σ remains unchanged.
(5) The time between occurrences of the assignable cause is exponentially distributed with a mean $1/\lambda$.
(6) When the process goes out of control, it stays out of control until detected and corrected.
(7) During each sampling interval, there exists at most one assignable cause which makes the process out of control. The assignable cause will not occur at sampling time.
(8) The measurement error is assumed to be zero.
(9) The cost function developed by Lorenzen and Vance (1986) is applied in the present paper to be the objective function for the economic design of the ARMA control chart. The expected cost (EC) per hour derived by Lorenzen and Vance (1986) includes the quality cost during production, the cost of false alarm, the cost of searching assignable cause, the cost of process correction, and the cost of sampling and inspection, and is mathematically expressed by

$$\mathrm{EC} = \frac{\frac{c_0}{\lambda} + c_1(nE + h\cdot\mathrm{ARL}_1 - \tau + r_1 t_1 + r_2 t_2) + \frac{s(Y+\gamma Q)}{\mathrm{ARL}_0}}{\frac{1}{\lambda} + (1-r_1)\frac{st_0}{\mathrm{ARL}_0} + nE + h\cdot\mathrm{ARL}_1 - \tau + t_1 + t_2} + \frac{(a+bn)\dfrac{\frac{1}{\lambda} - \tau + nE + h\cdot\mathrm{ARL}_1 + r_1 t_1 + r_2 t_2}{h} + W + \gamma Q}{\frac{1}{\lambda} + (1-r_1)\frac{st_0}{\mathrm{ARL}_0} + nE + h\cdot\mathrm{ARL}_1 - \tau + t_1 + t_2}, \tag{7}$$

where
n = sample size,
h = the sampling interval,
a = fixed cost per sample,
b = cost per unit sampled,
c_0 = quality cost per hour while producing in control,
c_1 = quality cost per hour while producing out of control ($c_1 > c_0$),
E = time to sample and chart one item,
Y = cost per false alarm,
Q = productivity loss per process cease,
W = cost to locate and correct the assignable cause,
s = expected number of samples taken while in control, and it may be shown that $s = \frac{e^{-\lambda h}}{1-e^{-\lambda h}}$,
τ = expected time of occurrence of the assignable cause between two samples while in control, and it is shown that $\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}$,
t_0 = expected search time when the signal is a false alarm,
t_1 = expected time to discover the assignable cause,