Expert Systems with Applications 39 (2012) 1848–1861

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Comparative analysis of SAW and TOPSIS based on interval-valued fuzzy sets: Discussions on score functions and weight constraints

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ARTICLE INFO

Keywords:
Interval-valued fuzzy set
Multiple-criteria decision analysis
SAW
TOPSIS
Computational experiment
Score function

ABSTRACT

Interval-valued fuzzy sets involve more uncertainties than ordinary fuzzy sets and can be used to capture imprecise or uncertain decision information in fields that require multiple-criteria decision analysis (MCDA). This paper takes the simple additive weighting (SAW) method and the technique for order preference by similarity to an ideal solution (TOPSIS) as the main structure to deal with interval-valued fuzzy evaluation information. Using an interval-valued fuzzy framework, this paper presents SAW-based and TOPSIS-based MCDA methods and conducts a comparative study through computational experiments. Comprehensive discussions have been made on the influence of score functions and weight constraints, where the score function represents an aggregated effect of positive and negative evaluations in performance ratings and the weight constraint consists of the unbiased condition, positivity bias, and negativity bias. The correlations and contradiction rates obtained in the experiments suggest that evident similarities exist between the interval-valued fuzzy SAW and TOPSIS rankings.

1. Introduction

The concept of interval-valued fuzzy sets (IVFSs) is defined by an interval-valued membership function (Sambuc, 1975; Zadeh, 1975), and an element's degree of membership in a set is characterized as a closed subinterval of [0, 1]. Because it may be difficult for decision-makers to exactly quantify their opinions of subjective judgments as a number within the interval [0, 1], it is better to represent the degree of membership by an interval rather than a single number. For this reason, IVFSs can be used to capture imprecise or uncertain decision information and many useful methods have been developed to enrich IVFS theory. Wang and Li (1998) defined interval-valued fuzzy numbers and interval-distributed numbers and provided a starting point for real-world applications. Deschrijver (2007) introduced some arithmetic operators for IVFSs. Vlachos and Sergiadis (2007) established a unified framework that includes the concepts of subsets, entropy, and cardinality for IVFSs. Wu and Mendel (2007) provided definitions of the centroid, cardinality, fuzziness, variance, and skewness of interval type-2 fuzzy sets. Zeng and Guo (2008) proposed a new axiomatic definition of the IVFS inclusion measure and examined relationships among the normalized distance, similarity measure, inclusion measure, and entropy of IVFSs. Sun, Gong, and Chen (2008) defined an interval-valued relation and built an interval-valued fuzzy information system. Bustince, Barrenechea, Pagola, and Fernandez (2009) presented a method for constructing IVFSs (or interval type-2 fuzzy sets) from a matrix (or image) and analyzed the application of IVFSs to edge detection in grayscale images. Bigand and Colot (2010) proposed a new fuzzy image filter, controlled by IVFSs, to remove noise from images. Yakhchali and Ghodsypour (2010) addressed the determination of possible values of the earliest and latest starting times of an activity in an interval-valued network with minimal time lag. Lu, Huang, and He (2010) developed an interval-valued fuzzy linear-programming method based on infinite α-cuts, and they applied this method to water resource management.

IVFSs involve more uncertainties than ordinary fuzzy sets. They allow for additional degrees of freedom to represent the uncertainty and fuzziness of the real world (Chen & Lee, 2010). Because IVFS theory is valuable in modeling imprecision and due to its ability to easily reflect the ambiguous nature of subjective judgments, IVFSs are suitable for capturing imprecise or uncertain information in fields that require multiple-criteria decision analysis (MCDA). Wei, Wang, and Lin (2011) introduced a correlation and correlation coefficients for interval-valued intuitionistic fuzzy sets. They then established an optimization model based on the negative ideal solution and max-min operator to solve multiple-attribute decision-making problems. Ye (2009) proposed a novel accuracy function for interval-valued intuitionistic fuzzy sets and applied weighted arithmetic average operator in MCDA. Yang, Lin, Yang, Li, and Yu (2009) combined IVFSs and soft sets to obtain an interval-valued fuzzy soft set. They defined the complement and the "and" and "or" operations, proved DeMorgan's associative and

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doi:10.1016/j.eswa.2011.08.065

distribution laws, and applied these to a decision-making problem. Ashtiani, Haghighirad, Makui, and Montazer (2009) presented an interval-valued fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) for solving MCDA problems. Wei and Chen (2009) applied their proposed similarity measure between interval-valued fuzzy numbers to develop a new fuzzy risk analysis algorithm. Chen and Chen (2009) presented a fuzzy risk analysis method based on a similarity measure between interval-valued fuzzy numbers and interval-valued fuzzy number arithmetic operators. Chen and Lee (2010) presented an interval type-2 fuzzy TOPSIS method to handle fuzzy multiple-attribute group decision-making problems based on interval type-2 fuzzy sets. To aggregate interval-valued intuitionistic fuzzy information, Xu (2010) proposed correlated averaging and geometric operators for interval-valued intuitionistic fuzzy processes. In the context of interval-valued intuitionistic fuzzy sets, Li (2010a) constructed a pair of nonlinear fractional programming models to calculate the relative closeness coefficient intervals of alternatives to the ideal solutions. In a similar manner, Li (2010b) developed TOPSIS-based nonlinear-programming methodology.

As a whole, the above-mentioned studies have focused on the extended simple additive weighting (SAW) or TOPSIS methods underlying interval-valued fuzzy information. The SAW method (Harsanyi, 1955) is a commonly known and very widely used method for providing a comparative evaluation procedure in MCDA. SAW uses all criterion values of an alternative and employs the regular arithmetical operations of multiplication and addition. Further, it is also necessary to determine a reasonable basis on which to form the weights reflecting the importance of each criterion. Einhorn and McCoach (1977) investigated the properties of SAW, including conditionally monotonic with utility and risk neutrality of the decision behavior. On the other hand, TOPSIS, developed by Hwang and Yoon (1981), is a well-known MCDA method. The basic concept of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. TOPSIS assumes that each criterion takes either monotonically increasing or monotonically decreasing utility. Both SAW and TOPSIS require the same input data and they can lead to a unique choice by comparing overall evaluations in SAW or closeness coefficients in TOPSIS.

In the decision context of IVFSs, substantial research took the SAW method or TOPSIS technique as the main structure to deal with multi-criteria evaluation information and to construct a priority ranking for a best alternative. The advantage of SAW is simple and easy to use and understand, while TOPSIS considers positive and negative ideal solutions as anchor points to reflect the contrast of the currently achievable criterion performances. Using an interval-valued fuzzy framework, the purpose of this study is to separately establish two MCDA methods using SAW and TOPSIS and then conduct a comparative study through computational experiments. Additional discussions have been made on the influence of score functions and weight constraints. First, a series of score functions for interval-valued evaluations is proposed from various perspectives to identify the mixed results of the outcome expectations. Based on the score functions, the degree of suitability to which each alternative satisfies the decision-maker's requirements, or instead, the relative degree of closeness of each alternative with respect to the positive ideal solution is defined. Because the information available on the relative importance of the multiple criteria for decision-making is often incomplete, this study proposes several optimization models with suitability functions or closeness coefficients for ill-known membership grades. To cope with different weight constraints of criterion importance, an integrated programming model is developed, utilizing both deviation variables and weighted suitability functions (or closeness coefficients). Furthermore, objective information in the decision matrix and subjective information of the criterion importance are combined to construct procedural steps using the SAW and TOPSIS methods for acquiring optimal decisions. Finally, a large set of random MCDA problems are generated, and computational studies are undertaken to compare preference orders determined by interval-valued fuzzy SAW and TOPSIS methods with several score functions and different conditions for the criterion weights.

2. Decision environment and weight assessment

Definition 1. Let Int([0, 1]) stand for the set of all closed subintervals of [0, 1]. Let X be an ordinary finite non-empty set. An IVFS A in X is an expression given by:

$$A = \{\langle x, M_A(x)\rangle | x \in X\}, \tag{1}$$

where the function M_A: $X \to$ Int([0, 1]) defines the degree of membership of an element x in A, such that $x \to M_A(x) = [M_A^-(x), M_A^+(x)]$.

Definition 2. For each IVFS A in X, the value of

$$W_A(x) = M_A^+(x) - M_A^-(x) \tag{2}$$

represents the width of the interval $M_A(x)$. $W_A(x)$ can be considered as the degree of uncertainty (or indeterminacy) or the degree of hesitancy associated with the membership of element $x \in X$ in IVFS A. Let $IVFS(X)$ denote the class of IVFSs in the universe X.

2.1. Decision matrix based on IVFSs

In the work presented here, evaluations of each alternative in an MCDA problem with respect to each criterion of the fuzzy concept "excellence" are given using IVFSs. Suppose that there exists a non-dominated set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$. Each alternative is assessed on n criteria, which are denoted by $X = \{x_1, x_2, \ldots, x_n\}$.Let M_{ij}: $X \to$ Int([0, 1]) such that $x_j \to M_{ij} = [M_{ij}^-, M_{ij}^+]$, where M_{ij}^- and M_{ij}^+ are the lower extreme and upper extreme, respectively, of the membership degrees of the alternative $A_i \in A$ with respect to the criterion $x_j \in X$ for the fuzzy concept "excellence." In addition, let $X_{ij} = \{\langle x_j, [M_{ij}^-, M_{ij}^+]\rangle\}$. The degree of uncertainty in alternative A_i in the set X_{ij} is defined by $W_{ij} = M_{ij}^+ - M_{ij}^-$. The interval-valued decision matrix D is defined in the following form:

$$D = \begin{bmatrix} & x_1 & x_2 & \cdots & x_n \\ A_1 & [M_{11}^-, M_{11}^+] & [M_{12}^-, M_{12}^+] & \cdots & [M_{1n}^-, M_{1n}^+] \\ A_2 & [M_{21}^-, M_{21}^+] & [M_{22}^-, M_{22}^+] & \cdots & [M_{2n}^-, M_{2n}^+] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & [M_{m1}^-, M_{m1}^+] & [M_{m2}^-, M_{m2}^+] & \cdots & [M_{mn}^-, M_{mn}^+] \end{bmatrix}, \tag{3}$$

where the characteristics of the alternative A_i can be represented by the IVFS as follows:

$$\begin{aligned} A_i &= \{\langle x_1, [M_{i1}^-, M_{i1}^+]\rangle, \langle x_2, [M_{i2}^-, M_{i2}^+]\rangle, \ldots, \langle x_n, [M_{in}^-, M_{in}^+]\rangle\} \\ &= \{\langle x_j, [M_{ij}^-, M_{ij}^+]\rangle | x_j \in X\}. \end{aligned} \tag{4}$$

In a similar manner, the decision-maker's weight lies in the closed interval $[w_j^l, \ w_j^u]$, where $0 \leqslant w_j^l \leqslant w_j^u \leqslant 1$ for each criterion $x_j \in X$. Because there is no objection in the literature to considering normalized weights, the criterion weights should be normalized to sum to one in general. Therefore, $\sum_{j=1}^n w_j^l \leqslant 1$ and $\sum_{j=1}^n w_j^u \geqslant 1$ are required to determine the weights $w_j \in [0, 1]$ $(j = 1, 2, \ldots, n)$ that satisfy $w_j^l \leqslant w_j \leqslant w_j^u$ and $\sum_{j=1}^n w_j = 1$.