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Handling partial truth on type-2 similarity-based reasoning

Chung-Ming Own *

Department of Computer and Communication Engineering, St. John's University, Tamsui, Taipei 62107, Taiwan, ROC

Abstract

Representation and manipulation of the vague concepts of partially true knowledge in the development of machine intelligence is a wide and challenging field of study. How to extract of approximate facts from vague and partially true statements has drawn significant attention from researchers in the fuzzy information processing. Furthermore, handling uncertainty from this incomplete information has its own necessity. This study theoretically examines a formal method for representing and manipulating partially true knowledge. This method is based on the similarity measure of type-2 fuzzy sets, which are directly used to handle rule uncertainties that type-1 fuzzy sets cannot. The proposed type-2 similarity-based reasoning method is theoretically defined and discussed herein, and the reasoning results are applied to show the usefulness with the comparison of the general fuzzy sets.

Keywords: Approximate reasoning; Type-2 fuzzy sets; Type-2 similarity; Partial truth

0. Introduction

Since much knowledge about many things and events in our daily life is necessarily incomplete, and since exceptions cannot be explicitly stated, people seek a mechanism that allows them to reach conclusions from incomplete information. Hence, they consistently reason approximately about incomplete knowledge with reasonably good results. Many approximate reasoning methods have been proposed. Zadeh (1965) first proposed the concept of approximate reasoning based on the theory of fuzzy sets and fuzzy logic. By Zadeh's assumption, the imprecision in natural language implies possibility. So far, fuzzy approximate reasoning has applied extensively in various soft sciences such as biology, and psychology, in design of expert systems, in risk analysis and in areas of advanced technology (Lee, 1990; Mizumoto, 1985; Zadeh, 1970).

According to (Raha & Ray, 1999), some commonsense knowledge representations cannot be taken for granted but are partially true. That is, knowledge about exceptions may be incomplete, and so the set of exceptions cannot be denoted exclusively by using a default rule. Therefore, reasoning systems should be modified to accommodate such partially true knowledge. Previous surveys have used fuzzy sets to represent and manipulate such knowledge (Raha & Ray, 1997, 1999, 2000; Zwick, Carlstein, & Budescu, 1987).

In addition, According to the general inference methods in the type-1 fuzzy reasoning systems, there are several existing deficiencies. The process heavily relies on the computation of the conditional relation and the given fact in the computation of a meaningful conclusion (Tsang, Lee, & Yeung, 1995). In consonance, Zadeh's max–min compositional inference rules have been widely criticized as too complicated with unclear semantics. Hence, in (Raha & Ray, 1997, 1999, 2000; Turksen & Zhong, 1988), they proposed similar methods as similarity-based reasoning. In such similarity-based reasoning methods, the desired conclusion is derived only using the similarity measure between the fact and the antecedent in a rule-based system. The conclusion becomes independent of the relational operator.

On the other hand, type-2 fuzzy sets, which were systematically proposed recently, are very appropriate for several applicable problems, including signal processing problems (Mendel, 2000). Type-2 fuzzy sets can effectively denote

* Tel.: +886 2 28013131x6328.
E-mail address: cmown@mail.sju.edu.tw

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and manage uncertain information. Meanwhile, much work has already been gone into defining the degree of similarity between two standard fuzzy sets. However, the similarity between type-2 fuzzy sets has received little attention, and the historical similarity measures cannot be directly applied to measure type-2 fuzzy sets. Till in 2004, Hung and Yang (2004) proposed the axiom definition for a type-2 similarity measure based on Hausdorff distance. They compute the similarities by treating the secondary grades as having two opposite elements consisting of coexistence on peer secondary membership functions. Mitchell (2005a) introduced the similarity measure for measuring the similarity between type-2 fuzzy sets, and applied the proposed method to solve the problem of automatic evaluation of welded structures. Mitchell (2005b) also defined the correlation coefficient between two type-2 fuzzy sets, and adopted the embedded function model to interpret each type-2 fuzzy sets as a weighted ensemble of fuzzy sets. Besides, Mitchell (2006) adopted a statistical viewpoint and interpreted each type-2 fuzzy number to rank uncertainty for each intuitionistic fuzzy number. Lee, Lee, and Lee (2005) proposed a comparison method for discrete type-2 fuzzy values based on the concept of the satisfaction function. Their method focused on the handling the ambiguity in fuzzy comparisons.

In this study, such partially true knowledge is represented and manipulated using the type-2 fuzzy sets firstly. Thus, the author models the partially true knowledge as uncertainties in rule-base logic systems, and theoretically examines a formal method to represent the partially true statement as higher membership value in type-2 fuzzy sets. Accordingly, to consist with the advantage of similarity-based reasoning, the author formulates an approach to measure the similarity between type-2 fuzzy sets based on the improvement of Hung and Yang (2004) method, and proposes a type-2 similarity-based reasoning method as another type-2 inference method.

Section 1 introduces the representation of partially true knowledge. Section 2 briefly reviews essential concepts of type-2 fuzzy sets. Section 3 describes the proposed reasoning method with similarity between type-2 fuzzy sets. Section 4 presents an empirical example to illustrate the utility of the proposed models. Section 5 draws conclusions.

1. Representation of partially true knowledge

Reasoning systems are altered to deal with incomplete or partially true knowledge by splitting each such partially true statement into two components – a proposition component and an associated truth value component (Dubois & Prade, 1990). The truth value component provides a mechanism for modifying the significance of an original proposition. Generally, a proposition such as "x is F" is expressed as "x is F is τ", where τ denotes a linguistic value of partial truth qualification, defined as the degree of compatibility of the situation with the proposition "x is F". Therefore, "x is F" is defined as a proposition component, and a linguistic truth value, τ, is defined as an associated truth value component. Equivalent statements in natural language include:

" 'David is healthy' is quite true."

" 'The speed is moderate' is absolutely true."

The unit interval $[0,1]$ is taken as the set of partially true values. Therefore, any vague definition related to truth can be represented by a fuzzy set on $[0,1]$.

A simple vague proposition about truth value, such as "This truth value represents 'mostly true'", can be translated into a rule of the form of general fuzzy sets,

$$\begin{aligned}\text{"Truth value is mostly true"} &= \sum_{x\in X}\mu_{\text{mostly true}}(x)/x\\ &= 0.4/0.65 + 0.5/0.7\\ &\quad + 0.75/0.75 + 1/0.8\\ &\quad + 0.75/0.9 + 0.26/0.95.\end{aligned}$$

The value of $\mu_{\text{mostly true}}(x)$ does not change the meaning of the proposition, but represents the subjective opinion about the meaning of the proposition. That is, when $\mu_{\text{mostly true}}(x)=0$, the truth value certainly differs from x, and when $\mu_{\text{mostly true}}(x)=1$, the truth value equals x. Notably, $\mu_{\text{mostly true}}(x)$ reveals the uncertainty of the original knowledge.

Bellman and Zadeh assumed that this partial truth qualification is local, rather than absolute (Zadeh, 1979; Bellman & Zadeh, 1977). They obtained the true statement based on the partially true statement, and derived the corresponding fuzzy set as a representation of the statement. Baldwin proposed the implied statements, which consists the fuzzy truth value restricted to a Lukasiewicz logic implication related on a fuzzy truth space (Baldwin, 1979). Then, he applied set-theoretic considerations to obtain fuzzy truth value constraints on truth value restrictions from conditional fuzzy linguistic statements, by applying an inverse procedure for modifying truth functions.

Accordingly, Raha and Ray (1997, 1999, 2000) proposed a method for reasoning with the partial truth value associated with a vague sentence. The partial truth values are defined by fuzzy sets on the universe of discourse $[0,1]$, which is a unit interval. This vague proposition relates to a possibility distribution, such that each possibility distribution is assigned to and manipulated by a fuzzy set/relation accordingly.

The approach applied in this study differs from the above approaches. This study attempts to eliminate the deficiencies of the representation of partial true knowledge. Although previous methods, associate and manipulate the partial truth value in accordance to the proposition, they denote and compute the corresponding fuzzy set and partial truth qualification separately. That is, set-theoretic considerations cannot be used to derive partial true knowledge theoretically, and partial truth statements are not