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# Parameter identification of chaotic systems using evolutionary programming approach

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**Abstract**

This paper is concerned with the parameter identification problem for chaotic systems. An evolutionary programming (EP) approach is newly introduced to solve this problem. The unknown parameters of chaotic systems are taken as a parameter vector, and will be optimally approximated to the exact values of parameters by using the proposed EP algorithm. The unified chaotic systems including Lorenz, Lü and Chen systems are used in an illustrative example to show the validity of the proposed method.


*Keywords:* Identification; Evolutionary programming approach; Unified chaotic system

## 1. Introduction

Much of modern physics can be described by ordinary differential equations. However, many interesting systems exist for which the exact parameters in ordinary differential equations are unknown, and those are either highly nonlinear or chaotic. Examples of such systems are physiological processes such as the electrical activity in the brain during epileptic seizures, and voice generation, etc.

On the other hand, the study of chaotic systems is motivated by the fact that many interesting natural and man-made phenomena, such as earthquakes, laser systems, and epileptic seizures are with chaotic nature. These phenomena have previously been thought to be stochastic, and therefore unpredictable. However, it has been revealed that it is indeed possible to predict time series generated by chaotic systems if the model of chaotic systems can be exactly constructed. Therefore, it follows an interesting problem in chaos theory; that is the identification or reconstruction of the unknown variables and parameters of a chaotic system (Al-Assaf, El-Khazali, & Ahmad, 2004). This problem is important because any experimental dynamical system possesses only a few variables or parameters that can be measured. In some cases, the non-available quantities are necessary to be estimated or to be reconstructed in order to completely determine the system's model and to achieve a good synchronization or predict the behavior of a system (Suarez-Castañon, Aguilar-Ibáñez, & Flores-Ando, 2003).

Recently, evolutionary programming (EP) algorithms have been become available and promising techniques for global optimization of complex functions, and have been applied to difficult search problems (Cao, 1997; Chen & Huang, 2003; Fogel, 1995; Huang, Tzeng, & Ong, 2006; Kim, Min, & Han, 2006; Ma & Wu, 1995; Yan, Hung, & Liao, 2007). This type of algorithm is modeled on processes found in natural evolution. The process of evolution inevitably leads to the optimization of ''behavior'' within the context of a given criterion. Consequently, the EP algorithm to search global optimization contains four main parts: initialization, mutation, competition, and reproduction.

Motivated by the aforementioned studies, this paper aims to present a method to identify the parameter of chaotic systems. Based on an extended EP algorithm, we will

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solve the parameter identification problem for chaotic systems.

The rest of this paper is organized as follows: a general form of chaotic system is addressed in Section 2. In Section 3, an evolutionary programming (EP) algorithm to solve the optimization problem is presented in detail. In Section 4, a numerical simulation for general unified chaotic systems is included to verify the feasibility of the proposed method. Finally, a brief conclusion is stated in Section 5.

## 2. Chaotic system description and problem formulation

Chaos, an interesting phenomenon in nonlinear dynamical systems, has been developed and thoroughly studied over the past two decades. Generally, chaotic systems are nonlinear deterministic systems those display complex, noisy-like and unpredictable behaviors. The sensitive dependence upon an initial condition and on the system's parameter variations is a prominent characteristic of chaotic behavior. In 1963, the Lorenz system is first proposed to model the unpredictable behavior of the weather. Recently, it has been revealed that the chaotic equations may describe several different engineering systems such as disk dynamos, laser devices and several problems related to convection (Richter, 2001).

In order to explore the problem in this paper, a general form for chaotic systems is given as follows:

General chaotic system:

$$\dot{x} = f(x, \Theta), \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$ is the state vectors and $\Theta = [\theta_1, \theta_2, \ldots, \theta_m]^{\mathrm{T}} \in R^m$ is the unknown parameters vector. $f: R^n \times R^m \to R^n$ is a given nonlinear vector function. Generally many chaotic systems, such as Lorenz system, Lü system, Chen system, Rössler system and Chua's circuit, etc., can be expressed by (1).

In order to estimate the unknown parameters in (1), a parameter identification system is defined below:

$$\dot{\hat{x}} = f(\hat{x}, \hat{\Theta}), \tag{2}$$

where $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^{\mathrm{T}}$ and $\hat{\Theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m]^{\mathrm{T}} \in R^m$ are of the estimated state vector and the estimated parameter vector, respectively, of the identification system.

To cope with the parameter identification problem for the general chaotic systems described in (1), the integrated absolute error (IAE) index is used as the objective function (OF), which is defined as

$$\mathrm{OF} := \mathrm{IAE} = \int_0^\infty \|E(\tau)\| \mathrm{d}\tau, \tag{3}$$

where $E(t) = [e_1 \quad e_2 \quad \cdots \quad e_n]^{\mathrm{T}} = [x_1 - \hat{x}_1 \quad x_2 - \hat{x}_2 \quad \cdots \quad x_n - \hat{x}_n]^{\mathrm{T}}$, and $\|\cdot\|$is the Euclidean norm of a vector. The problem considered in this paper is that for the same initial conditions of systems (1) and (2), a parameter vector $\hat{\Theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots \hat{\theta}_m]^{\mathrm{T}}$ can be obtained such that the objective function in (3) can be minimized as possible and system parameters can be appropriately identified.

## 3. An evolutionary programming (EP) algorithm to solve the optimization problem

Since the EP algorithm has been considered as a promising technique for the global optimization of complex functions, we introduce the EP algorithm to solve this parameter identification problem. Virtually, there is one important factor: the population size to be necessarily considered when the EP algorithm is utilized. It begins with generating a population of parameter vector whose initial values are uniformly and randomly generated from the user-defined intervals$[\hat{\theta}_{\min}, \hat{\theta}_{\max}]$, where $\hat{\theta}_{\min}$ and $\hat{\theta}_{\max}$ stand for the lower bound and upper bound of searching space, respectively, during evolutions. Furthermore, the quasi-random sequence (QRS) is used to generate the initial population for EP to avoid causing clustering around an arbitrary local optimal (Cao, 1997). The optimization problem is to find the best parameter vector $\hat{\Theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m]^{\mathrm{T}}$ such that the value of IAE in (3) is minimized as possibly.

### 3.1. Optimization problem formulation

Let $\hat{\Theta}$ be the continuously differentiable matrix-valued function defined for $\hat{\Theta} \in S$, where $S = \{\hat{\Theta} \in R^m \mid \hat{\theta}_{\min} \leqslant \hat{\theta}_j \leqslant \hat{\theta}_{\max}, \hat{\theta}_{\min} < \hat{\theta}_{\max} < \infty, j = 1, 2, 3 \ldots m\}$. The optimization problem considered here is to find $\hat{\Theta}^* = [\hat{\theta}_1^*, \hat{\theta}_2^*, \hat{\theta}_3^* \cdots \hat{\theta}_m^*]^{\mathrm{T}} \in S$ such that the performance index of IAE (3) is minimized. More precisely, the optimization problem can be described mathematically as follows: To find $\hat{\Theta}^* \in S$ such that

$$\mathrm{IAE} = \int_0^\infty \|E(\tau)\| \mathrm{d}\tau \quad \text{for } \hat{\Theta}^* \in S \tag{4}$$

is minimized.

### 3.2. Optimization procedures

The procedures of EP algorithm for solving the above optimal parameter identification problem are described as follows:

(I) According to the quasi-random sequence (QRS), generate an initial population $p_0 = [p_1, p_2, \ldots, p_H]$ of size $H$ by randomly initializing each $m$-dimensional solution vector $p_\alpha \in S, \alpha = 1, 2, \ldots, H$.

(II) Calculate the fitness score IAE, $g_\alpha = g(p_\alpha)$ for $p_\alpha$ $\alpha = 1, 2 \ldots, H$, where $g(p_\alpha) = \int_0^\infty \|E(\tau)\| \mathrm{d}\tau$, $p_\alpha = [\hat{\theta}_1 \quad \hat{\theta}_2 \quad \hat{\theta}_3 \quad \cdots \quad \hat{\theta}_m]^{\mathrm{T}} \in S$.

(III) Generate $p_{\alpha+H}$ by mutating $p_\alpha$, $\alpha = 1, \ldots, H$, to double the population size from $H$ to $2H$ in the following ways:

$$p_{\alpha+H,\gamma} = p_{\alpha,\gamma} + N\left(0, \beta \frac{g_\alpha}{g_\Sigma}\right) \quad \forall \gamma = 1, 2, 3 \ldots m, \tag{5}$$

where $p_{\alpha,\gamma}$ denotes the $\gamma$th element of the $\alpha$th individual, $N(0, \beta \frac{g_\alpha}{g_\Sigma})$ represents a Gaussion random variable with a mean zero and variance $\beta \frac{g_\alpha}{g_\Sigma}$, $g_\Sigma$ represents the