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ScienceDirect
Fuzzy Sets and Systems 300 (2016) 147–151
FUZZY sets and systems
www.elsevier.com/locate/fss

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Note on stratified L-ordered convergence structures ☆

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Received 24 July 2015; received in revised form 31 December 2015; accepted 11 January 2016

Available online 21 January 2016

Abstract

In the paper, we point out that the proof of Theorem 5.5 in Fang (2010) [1], which says the category of stratified L-ordered convergence spaces is Cartesian-closed, is not correct. By an alternative method, the Cartesian-closedness of the category of stratified L-ordered convergence spaces is confirmed.

Keywords: L-filter; L-ordered convergence structure; Cartesian-closedness; Topology

1. Introduction

When the underlying lattice is a complete Heyting algebra L, G. Jäger [5] developed a theory of stratified L-generalized convergence spaces based on the concept of stratified L-filters. The resulting category *SL*-**GCS** of stratified L-generalized convergence spaces contains the category of stratified L-topological spaces as a reflective subcategory and has the desired structural property of Cartesian-closedness.

J.M. Fang [1] proposed the concept of stratified L-ordered convergence structures by making use of the intrinsic fuzzy inclusion order on the fuzzy power set. Note that Li [6] also obtained this concept using a different name, called stratified L-convergence structures. Fang in [1] showed that the category of stratified L-ordered convergence spaces *SL*-**OCS** is a reflective full subcategory in *SL*-**GCS**. Further Fang [2] observed there is a Galois correspondence between *SL*-**OCS** and the category of strong L-topological spaces [8].

Following the property of a reflective full subcategory in *SL*-**GCS**, in order to answer the Cartesian-closedness of *SL*-**OCS**, one of the ideas is to prove that the reflector $(-)_* : SL\text{-}\mathbf{GCS} \to SL\text{-}\mathbf{OCS}$ preserves finite products. This is precisely the strategy in the proof of Theorem 5.5 [1], which says that the category *SL*-**OCS** is Cartesian-closed.

DOI of original article: http://dx.doi.org/10.1016/j.fss.2010.04.001.
☆ This work is supported by Natural Science Foundation of China (No. 11471297, No. 11401547).
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http://dx.doi.org/10.1016/j.fss.2016.01.002
0165-0114/

However, we find that the method of proving that the reflector $(-)_* : SL\text{-}\mathbf{GCS} \to SL\text{-}\mathbf{OCS}$ preserves finite products is not correct. In detail, the inequality at line 5 of p. 2148 in [1] is wrong.

The aim of this note is to offer a solution to confirm the Cartesian-closedness of SL-**OCS**.

2. Preliminaries

To explore the solution, let us make some necessary preliminaries.

Throughout this paper, L denotes a complete Heyting algebra. The greatest element of L is denoted by 1 and the least element of L by 0. In a complete Heyting algebra L, an implication can be defined by $a \to b = \bigvee\{c \mid a \wedge c \leq b\}$. This implication is a right-adjoint of the meet operation, i.e. it satisfies $c \leq a \to b$ if and only if $a \wedge c \leq b$. For the properties of an implication, we refer readers to [5].

An L-subset on a set X is a map from X to L, and the family of all L-subsets on X will be denoted by L^X, called the L-power set. By 0_X and 1_X, we denote the constant L-subsets on X taking the value 0 and 1, respectively. For $A, B \in L^X$, the degree to which B contains A is denoted by $\mathcal{S}(A, B)$, which is given by $\mathcal{S}(A, B) = \bigwedge_{x\in X}(A(x) \to B(x))$.

Definition 2.1. (See [3,4].) Let L be a Heyting algebra and X be a nonempty set. A map $\mathscr{F} : L^X \to L$ satisfies the following conditions: for all $A, B \in L^X$, $\alpha \in L$,

(F1) $\mathscr{F}(1_X) = 1$, $\mathscr{F}(0_X) = 0$,
(F2) $A \leq B \Rightarrow \mathscr{F}(A) \leq \mathscr{F}(B)$,
(F3) $\mathscr{F}(A) \wedge \mathscr{F}(B) \leq \mathscr{F}(A \wedge B)$,
(Fs) $\alpha \wedge \mathscr{F}(A) \leq \mathscr{F}(\alpha \wedge A)$,

then $\mathscr{F}$ is called a stratified L-filter. The set of all stratified L-filters is denoted by $F_L^s(X)$, order it by $\mathscr{F} \leq \mathscr{G}$ if $\mathscr{F}(A) \leq \mathscr{G}(A)$ for all $A \in L^X$. □

Example 2.2. (See [4].) Given a point $x \in X$, the map $[x] : L^X \to L$ defined by for each $A \in L^X$, $[x](A) := A(x)$ is a stratified L-filter on X, called it the point L-filter $[x]$ of x. □

Let $\varphi : X \to Y$ be a map and $\mathscr{F} \in F_L^s(X)$. Then the map $\varphi^{\Rightarrow}(\mathscr{F}) : L^Y \to L$, defined by $\varphi^{\Rightarrow}(\mathscr{F})(B) = \mathscr{F}(B \circ \varphi)$ for all $B \in L^Y$, is a stratified L-filter, called the image of $\mathscr{F}$ under φ in [3]. For a nonempty set X, a binary map $\mathcal{S}_F(-, -) : F_L^s(X) \times F_L^s(X) \to L$ is defined in [1] by

$$\forall \mathscr{F}, \mathscr{G} \in F_L^s(X),\ \mathcal{S}_F(\mathscr{F}, \mathscr{G}) = \bigwedge_{A\in L^X} (\mathscr{F}(A) \to \mathscr{G}(A)).$$

Obviously, if $\varphi : X \to Y$ is an ordinary map, then for $\mathscr{F}, \mathscr{G} \in F_L^s(X)$ it follows that $\mathcal{S}_F(\mathscr{F}, \mathscr{G}) \leq \mathcal{S}_F(\varphi^{\Rightarrow}(\mathscr{F}), \varphi^{\Rightarrow}(\mathscr{G}))$.

Let $\mathscr{F}$ and $\mathscr{G}$ be stratified L-filters on X and Y, respectively. The product of $\mathscr{F}$ and $\mathscr{G}$, denoted by $\mathscr{F} \times \mathscr{G}$, is defined by Jäger in [5]. We only offer one of its computations in the following proposition.

Proposition 2.3. *(See J.M. Fang [1].) Let $\mathscr{F} \in F_L^s(X_1)$, $\mathscr{G} \in F_L^s(X_2)$. The product filter $\mathscr{F} \times \mathscr{G}$ can be computed as follows (see Proposition 3.11 [1]):*

$$\forall A \in L^{X_1\times X_2},\ \big(\mathscr{F} \times \mathscr{G}\big)(A) = \bigvee_{C\in L^{X_1}, D\in L^{X_2}} \mathscr{F}(C) \wedge \mathscr{G}(D) \wedge \mathcal{S}(C \times D,\ A). \qquad \square$$

Lemma 2.4. *Let X and Y be two nonempty sets. Then for $\mathscr{F}, \mathscr{G} \in F_L^s(X)$ and $\mathscr{H} \in F_L^s(Y)$, $\mathcal{S}_F(\mathscr{F} \times \mathscr{H}, \mathscr{G} \times \mathscr{H}) \geq \mathcal{S}_F(\mathscr{F}, \mathscr{G})$.*

Proof. By Proposition 2.3, $\mathscr{F} \times \mathscr{H}$ and is computed by for $A \in L^{X\times Y}$,

$$\Big(\mathscr{F} \times \mathscr{H}\Big)(A) = \bigvee_{C\in L^X, D\in L^Y} \mathscr{F}(C) \wedge \mathscr{H}(D) \wedge \mathcal{S}(C \times D,\ A).$$