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Generalized ordered modular averaging operator and its application to group decision making

Jinpei Liu [a], Huayou Chen [b,*], Qin Xu [c], Ligang Zhou [b], Zhifu Tao [b]

[a] *School of Business, Anhui University, Hefei, Anhui, 230601, China*
[b] *School of Mathematical Sciences, Anhui University, Hefei, Anhui, 230601, China*
[c] *MOE Key Laboratory of Intelligence Computing and Signal Processing, Anhui University, Hefei, Anhui, 230601, China*

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Abstract

In this paper, we propose a new class of operators called the ordered quasi-modular averaging (OQMA) operators based on the ordered modular averaging (OMA) operators. It is shown that the OQMA operators are monotonic, bounded, idempotent, commutative and quasi-comonotone modular. Moreover, we introduce the generalized ordered modular averaging (GOMA) operator, which is a special case of the OQMA operator. Some special cases of the GOMA operator are discussed. An orness measure to reflect the or-like degree of the GOMA operator is proposed. We further extend the GOMA operator to the generalized ordered hybrid modular (GOHM) operator, which focuses not only on the degree of importance with respect to input arguments but also their serial positions. Finally, a new method based on the GOHM operator for multi-attribute group decision making is presented and a numerical example shows that the developed approach is feasible.

Keywords: Aggregation operator; OWA; Ordered quasi-modular averaging (OQMA) operator; Generalized ordered modular averaging (GOMA) operator; Orness; Multiple attribute group decision making

1. Introduction

Aggregation operators, which can aggregate several input arguments into a single representative output, have been employed in many fields [2–4,6,12,14,33,35], such as decision making, neural networks, pattern recognition, combination forecasting, supply chain management and statistical regression.

It is worth mentioning that Yager introduced the ordered weighted averaging (OWA) operator [29], which assigns the ith weight to the ith largest input value. A fundamental aspect of the OWA operator is a reordering step in which the input arguments are rearranged according to their values. Since then, the OWA operator has been studied

* Corresponding author. Tel.: +8 655 1638 61453.
E-mail addresses: liujinpei2012@163.com (J. Liu), huayouc@126.com (H. Chen), chinnymeers@163.com (Q. Xu), shuiqiaozlg@126.com (L. Zhou), zhifut_0514@163.com (Z. Tao).

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and applied by a lot of researchers, and various types of aggregation operators were proposed based on it. Among the different extensions of the OWA operator, we focus on the ordered weighted geometric (OWG) operator [24], the generalized OWA (GOWA) operator [31], the generalized Choquet aggregation operator, the ordered weighted maximum (OWMax) operator and the quasi-OWA (QOWA) operator [10]. Merigó and Gil-Lafuente proposed an induced GOWA (IGOWA) based on the GOWA operator [21], which is an extension of the induced OWA (IOWA) operator [34] and the induced ordered weighted geometric averaging (IOWGA) operator [7]. Besides, Power operator [28,30,36] and Bonferroni mean operator [1,26,27,32] can capture the interrelationships among the input arguments.

The orness measure was firstly proposed for power means under the name of disjunction degree [8], and the orness measure for the OWA operator was developed independently in [29], which is very important both in theory and applications. A lot of researchers have been focusing on the extension of Yager's orness concept to other aggregation operators. Marichal proposed the degrees of orness which can be defined for any compensative aggregation operator [19]. Yager defined the orness measure for GOWA operator in [31]. Salido and Murakami extended the OWA orness measure to fuzzy aggregation operators [23]. A general framework with the unipolar and bipolar parametric characterization of aggregation operators was introduced where the orness measure in the class of aggregation operators were discussed in [13]. Liu further proposed two new orness measures for the quasi-arithmetic means and the continuous weighted quasi-arithmetic mean respectively [16,17].

Recently, Mesiar and Mesiarová-Zemánková introduced the ordered modular averaging (OMA) operator [22], which is applied to deal with additive preference relations. However, it is unsuitable for aggregating decision making information taking the form of multiplicative preference relations. Based on the OMA operator, the ordered multiplicative modular geometric (OMMG) operator was developed [5]. However, the OMMG operator cannot satisfy idempotency.

In this paper, we firstly present the ordered quasi-modular averaging (OQMA) operators based on the quasi-modular aggregation operators, which extend the OMA operator and provide a very general formulation. Moreover, the GOMA operator is developed, which is a special case of the OQMA operator. We could point that the GOMA operator is monotonic, bounded, idempotent and commutative. Especially, when the controlling parameter λ and the associated functions f_i $(i = 1, 2, \dots, n)$ of the GOMA operators take different types, we can obtain a wide range of aggregation operators including the OMA operator, the OWG operator, the GOWA operator, the generalized Choquet integral and the OWMax operator, etc. An important point we would like to emphasize is that we can choose appropriate controlling parameter λ and the associated functions f_i $(i = 1, 2, \dots, n)$ to meet different practical requirements. Consequently, it is more feasible.

Moreover, we will present an orness measure of the GOMA operator, and study some properties of the GOMA operator associated with its orness measure. We further extend the GOMA operator to the generalized ordered hybrid modular (GOHM) operator, which focuses not only on the degree of importance with respect to input arguments but also their serial positions. Finally, a new method based on the GOMA operator and the GOHM operator for multi-attribute group decision making is presented and a numerical example shows that the developed approach is feasible.

This paper is organized as follows. In Section 2, we briefly review some basic concepts of the OWA operator, the OWG operator, the GOWA operator and the OMA operator. We develop the OQMA operators in Section 3. In Section 4, we introduce the GOMA operators and look at their properties. In Section 5, some special cases of the GOMA operators are investigated. An orness measure of the GOMA operator and the GOHM operator are developed in Section 6 and Section 7, respectively. A new approach for multi-attribute group decision making is presented and an illustrative example is given in Section 8. Section 9 ends the paper with concluding remarks.

2. Preliminaries

In this section, we shall briefly recall some fundamental concepts including the OWA operator [29], the OWG operator [24], the QOWA operator [11], the GOWA operator [31], the OMA operator [22] and the OMMG operator [5].

An aggregation operator [4] is a function of n $(n > 1)$ arguments $F : [0, 1]^n \to [0, 1]$, which should satisfy the following properties: (i) $F(0, 0, \dots, 0) = 0$ and $F(1, 1, \dots, 1) = 1$; (ii) $x \le y$ implies $F(x) \le F(y)$, for all $x, y \in [0, 1]^n$. If for any $x \in [0, 1]^n$, it is bounded by $\min_{i=1,2,\dots,n}(x_i) \le F(x_1, x_2, \dots, x_n) \le \max_{i=1,2,\dots,n}(x_i)$, then the aggregation operator is averaging, which imply that the averaging aggregation operator is idempotent, i.e., $F(x, x, \dots, x) = x$ for all $x \in [0, 1]$. In this paper, we will only discuss the averaging aggregation operators.