Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Sets and Systems 283 (2016) 56–82
FUZZY sets and systems
www.elsevier.com/locate/fss

Conditioned weighted L–R approximations of fuzzy numbers

Adrian I. Ban [a], Lucian Coroianu [a], Alireza Khastan [b]

[a] Department of Mathematics and Informatics, University of Oradea, 410087 Oradea, Romania
[b] Department of Mathematics, Institute for Advanced Studies in Basic Sciences, Zanjan 45137-66731, Iran

Received 11 September 2014; received in revised form 24 December 2014; accepted 22 March 2015

Available online 27 March 2015

Abstract

We compute the extended weighted L–R approximation of a given fuzzy number by a method based on general results in Hilbert spaces, the weighted average Euclidean distance being considered. The metric properties of the extended weighted L–R approximation of a fuzzy number are proved. We elaborate on a general method to study the existence, uniqueness and to calculate the L–R approximations of fuzzy numbers under the preservation of some parameters. We apply the results to find the weighted L–R approximations preserving ambiguity and value and respectively width in the general and unimodal case.

Keywords: Fuzzy number; L–R fuzzy number; Approximation

1. Introduction

Uncertain and incomplete information is often represented by fuzzy numbers in many practical areas (decision making, expert systems, pattern recognition, data mining, etc.).

The approximation of general fuzzy numbers by particular fuzzy numbers, with a simpler form, was intensively investigated in the last years (see, e.g., [4–14,32,33,35,42–49]). The first papers were dedicated to find the nearest interval, triangular or trapezoidal fuzzy number of a given fuzzy number, without an additional condition [1,27,37,42] or imposing the preservation of an important parameter [4,6,9,28–30,35,36,41,43,44], the average Euclidean distance between fuzzy numbers being considered. Later on, the topic has been developed by considering other distances between fuzzy numbers (weighted Euclidean distance, Hamming distance, source distance) and different kinds of fuzzy numbers (parametric, semi-trapezoidal, polynomial, piecewise linear) as approximations, with or without additional constraints [2,3,5,8,11,12,18,19,31,38,45,47,48].

The L–R fuzzy numbers were first introduced by Dubois and Prade and afterward used in applications and theoretical developments as well. In the very recent paper [48] the approximation of fuzzy numbers by L–R fuzzy numbers,

E-mail addresses: aiban@uoradea.ro (A.I. Ban), lcoroianu@uoradea.ro (L. Coroianu), Khastan@iasbs.ac.ir (A. Khastan).

http://dx.doi.org/10.1016/j.fss.2015.03.012

without preserving any parameter, with respect to weighted Euclidean distance was investigated. The calculus of the approximation was given and its properties were studied.

In the present paper we consider approximations of fuzzy numbers by L–R fuzzy numbers such that one or more parameters of the form $al_e + bu_e + cx_e + dy_e$ are preserved, where $a, b, c, d \in \mathbb{R}$ and l_e, u_e, x_e, y_e are the coordinates of the extended weighted L–R approximation. The weighted average Euclidean distance is considered and the results generalize many recent approximations with constraints. In Section 2 we present some preliminaries about fuzzy numbers and extended fuzzy numbers. The extended weighted L–R approximation of a fuzzy number is computed and its metric properties are studied in Section 3. The results of existence and uniqueness related to the weighted L–R approximations, with additional conditions in the form described above, are given in Section 4, by using theoretical aspects in Hilbert spaces. A simple method of computation of the weighted L–R approximation or weighted L–R unimodal approximation is proposed in Section 5. It is applied to two particular cases: weighted L–R approximation preserving ambiguity and value and weighted L–R approximation preserving width. Properties of the conditioned weighted L–R approximations of fuzzy numbers are discussed in Section 6.

2. Fuzzy numbers and extended fuzzy numbers

We consider the α-level representation of a fuzzy number A (as in [15] or [23]), that is

$$A_\alpha = [A_L(\alpha), A_R(\alpha)], \alpha \in [0,1],$$

where $A_L, A_R : [0,1] \to \mathbb{R}$ are bounded, left-continuous functions in $(0,1]$ and right continuous at 0 such that A_L is non-decreasing, A_R is non-increasing and $A_L(1) \le A_R(1)$. We denote by $F(\mathbb{R})$ the set of all fuzzy numbers. For $A, B \in F(\mathbb{R}), A_\alpha = [A_L(\alpha), A_R(\alpha)]$ and $B_\alpha = [B_L(\alpha), B_R(\alpha)]$, the quantity $d_\lambda(A, B)$ given by

$$d_\lambda^2(A,B) = \int_0^1 \lambda_L(\alpha)(A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 \lambda_R(\alpha)(A_R(\alpha) - B_R(\alpha))^2 d\alpha, \tag{1}$$

where λ_L and λ_R, which are called weight functions, are nonnegative functions such that $0 < \int_0^1 \lambda_L(\alpha) d\alpha < \infty$ and $0 < \int_0^1 \lambda_R(\alpha) d\alpha < \infty$, defines a weighted distance between A and B (see, e.g., [45]). If $\lambda_L(\alpha) = \lambda_R(\alpha) = 1$, for every $\alpha \in [0,1]$, then we get the well-known average Euclidean distance between A and B. Many other weight functions λ_L, λ_R and hence different weighted distances are proposed in the literature (see [33,49]).

The expected interval $EI(A)$ and the expected value $EV(A)$ of a fuzzy number A are given by (see [24,34])

$$EI(A) = \left[E_*(A), E^*(A)\right] = \left[\int_0^1 A_L(\alpha)d\alpha, \int_0^1 A_R(\alpha)d\alpha\right], \tag{2}$$

$$EV(A) = \frac{1}{2}(E_*(A) + E^*(A)). \tag{3}$$

The ambiguity $Amb(A)$, value $Val(A)$ and width $w(A)$ of a fuzzy number A are defined by (see [18,21,26])

$$Amb(A) = \int_0^1 \alpha(A_R(\alpha) - A_L(\alpha))d\alpha, \tag{4}$$

$$Val(A) = \int_0^1 \alpha(A_R(\alpha) + A_L(\alpha))d\alpha, \tag{5}$$

$$W(A) = \int_0^1 (A_R(\alpha) - A_L(\alpha))d\alpha, \tag{6}$$