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Fuzzy Sets and Systems 283 (2016) 83–107

FUZZY sets and systems

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Analytical fuzzy plane geometry III

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Received 31 October 2013; received in revised form 15 October 2014; accepted 17 October 2014

Available online 23 October 2014

Abstract

In this study, we attempt to construct *fuzzy circles* in a fuzzy geometrical plane. We provide a comprehensive study where we find a fuzzy number with a predetermined fuzzy distance from a given fuzzy number. We formulate fuzzy circles using the conventional basic definitions of crisp circles. Two different formulations of fuzzy circles are proposed, which depend on the information known about a fuzzy circle. The interrelations between the evaluated fuzzy circles are investigated. We show that the center of a fuzzy circle may not be a fuzzy point. However, the radius of a fuzzy circle that passes through three given fuzzy points is always a fuzzy number. The concepts of *same and inverse points* and a *fuzzy number along a line* are used to define and analyze the proposed ideas. Our discussions and studies are supported by suitable numerical and pictorial examples.

Keywords: Fuzzy circle; Fuzzy distance; Fuzzy number; Fuzzy number along a line; Fuzzy point; Same and inverse points

1. Introduction

In this study, we continue our investigation of analytical fuzzy plane geometry. In this series of studies on analytical fuzzy plane geometry [4,12], the basic paper [12] introduced the concepts of *same and inverse points* and a *fuzzy number along a line*. Using these concepts, in [12], we defined a fuzzy distance, a convex combination of fuzzy points, a fuzzy line segment, a fuzzy angle, and the containment of a fuzzy point in a fuzzy line segment. The next study [4] in the series defined and investigated *fuzzy lines*, and their various properties. Four different forms of fuzzy lines, i.e., a two-point form, a point-slope form, a slope-intercept form, and an intercept form, as well as their interrelations, were introduced in [4]. Formulations of a fuzzy line passing through several fuzzy points with collinear cores were also described in [4].

In the present study, we provide a comprehensive investigation of the construction of *fuzzy circles*. The fuzzy cone [1] is not well defined in previous studies, but the fuzzy circle may be considered as a fuzzy conic section. In fuzzy space geometry, we consider that the fuzzy cone may be visualized as a crisp cone with a noisy (imprecise) boundary;

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http://dx.doi.org/10.1016/j.fss.2014.10.018
0165-0114/© 2014 Published by Elsevier B.V.

indeed, its core must be a crisp cone. If this is the structure of a fuzzy cone, then the geometrical figures of fuzzy circles can be obtained by intersecting a double-napped fuzzy right circular cone with a crisp plane (not with a fuzzy plane [26]) perpendicular to the axis of the core of the cone.

The underlying approach to the proposed formulations of fuzzy circles essentially uses the conventional definitions [31] of crisp circles. Buckley and Eslami [2,3] used the sup-min composition of fuzzy sets directly to define fuzzy lines, fuzzy circles, etc. It was reported in [4,12] that the fuzzy lines and their various properties defined in [2] lose their inner conformity with the well-known basic definitions when they are reduced to crisp sets. The fuzzy circles defined in [3] also do not agree with the conventional geometrical idea that a 'circle is the locus of a point, which has a fixed distance from a fixed point.' The problem that arises from [2,3] may be due to the direct use of the sup-min composition for the well-known algebraic equations of lines, circles, etc., with the coefficients as fuzzy numbers. The membership functions that correspond to the definitions given in [2,3] are also very difficult to depict and it is rarely possible to find their closed forms. Thus, there is a need to construct a fuzzy circle using a different approach to fuzzifying the well-known algebraic equation of a circle. It was observed in [4,12] that instead of fuzzifying the well-known algebraic equations of fuzzy lines, we may generalize the original definitions and the membership functions can have better formulations with easier evaluations of the membership values than those described in [2]. In the present study, we attempt to construct fuzzy circles based on the original definition of the corresponding classical circles. First, we review previous studies of fuzzy conics before providing the details of the proposed formulation.

Apart from [3] and its explication [5], some ideas about fuzzy conics and hence fuzzy circles were developed and applied in [8,10,15–29,33]. Some concepts related to fuzzy geodesic spheres, fuzzy spheres, and fuzzy circles are given in [6,7,9,11].

In order to form a fuzzy quantitative circle, the conventional unit circle was modified in [24] by introducing quantity spaces for orientation and translation. The core of the constructed fuzzy unit circle in [24] is not a crisp unit circle. A fuzzy disk was defined in [29] where the membership function depended only on the distance from its center. This fuzzy disk is similar to a fuzzy point in [2]. In [22], fuzzy conics are defined by blurring their boundaries using a smooth unit step function and implicit functions. However, in the shapes obtained, their 1-level sets contain all the points that lie outside the conic instead of the points on the boundary of the desired conic. The core of the fuzzy circle used in [14] is not a crisp circle. Using a fuzzy inequality, Obradović et al. [25] defined a fuzzy circle. However, in [25], the center of a fuzzy circle is considered to be a crisp point. The fuzzy circles in [16,17] are basically the fuzzy points described in [2]. Savoj and Monadjemi [30] defined a fuzzy circle as a collection of several concentric circles with a continuous radius. The fuzzy circle in [30] may be a particular case of a general fuzzy circle.

Fuzzy conics are used in [10] as prototypes for fuzzy criterion clustering of a finite number of crisp or fuzzy data. The definitions considered are suitable, but they may not be treated as general definitions of fuzzy conics, since the center or foci of the fuzzy conics are taken as crisp points. However, in the general definition, a fuzzy point instead of a crisp point would have been more appropriate to represent a focus or center of a fuzzy conic.

In [33], Zadeh mentioned that the counterpart of a crisp circle C in Euclidean geometry is a fuzzy circle. A fuzzy circle can be formed by a fuzzy transform of C, where C is a prototype for the fuzzy circle. A fuzzy circle can be viewed as a fuzzy transformation of C drawn by a spray pen [33]. In this case, the fuzzy transformation is a one-to-many function.

We note that ideas of fuzzy circles have been proposed by various researchers, but previous studies have simply focused on the detailed construction of fuzzy circles. Only Buckley and Eslami studied concepts related to the construction of fuzzy circles [3]. In the present study, we propose new methodologies to define fuzzy circles. We focus on the detailed procedures for constructing fuzzy circles. Two methodologies are used to define fuzzy circles. In the first methodology, the basic definition of a circle is extended to the fuzzy environment using the same and inverse points, where we assume that its center is a fuzzy point and that its radius is a fuzzy number. In the second approach, a fuzzy circle is constructed that passes through three fuzzy points. Based on the second approach, we investigate the formulations and properties of the fuzzy center and fuzzy radius. The following is an outline of this study.

In Section 2, we provide a comprehensive investigation of how to find a fuzzy number (or a fuzzy number along a line), which is a fixed fuzzy distance from a given fuzzy number (or a fuzzy number along a line). The method presented in Section 2 is then used to define fuzzy circles. Section 3 considers fuzzy circles and their properties using two different approaches. The equivalence between the two methods is demonstrated in Section 4. In Section 4, we also compare the proposed fuzzy circle with the existing formulations of fuzzy circles. Concluding remarks and a discussion of the future scope of our research into fuzzy circles are given in Section 5.