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H_∞ controller design for affine fuzzy systems based on piecewise Lyapunov functions in finite-frequency domain

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Abstract

This paper studies the problem of state feedback controller design for a class of nonlinear systems described by continuous-time affine fuzzy models. Based on a piecewise continuous Lyapunov function combined with S-procedure and some matrix decoupling techniques, a novel control law is derived in the formulation of linear matrix inequalities (LMIs) in finite-frequency domain. First, a so-called finite-frequency H_∞ performance index is defined, which extends the standard H_∞ performance. Then, a sufficient condition is derived such that the fuzzy closed-loop system satisfies a finite-frequency H_∞ performance. By introducing the S-procedure and adding slack variables, piecewise controllers are designed to deal with disturbances in the low-, middle-, and high-frequency domain, respectively. The proposed piecewise controller design method can get a better disturbance-attenuation performance when the frequency ranges of disturbances are known beforehand. Finally, two examples are given to illustrate the effectiveness and superiority of the new results.

Keywords: Affine fuzzy models; Finite frequency; Piecewise Lyapunov function; Linear matrix inequalities (LMIs)

1. Introduction

The Takagi–Sugeno (T–S) fuzzy model of nonlinear systems has been focused a lot since it was proposed in 1985 [1]. Incorporating of linguistic information from human experts and "blending" of locally linear systems have proved that the T–S fuzzy model is a good universal approximator [2]. T–S fuzzy control systems have been extensively studied and a number of significant stability analysis results and controller synthesis methods have been obtained by taking full advantage of Lyapunov stability theory (see [3–9] and the references therein).

Recently, an increasing amount of work has been devoted to stability analysis and controller synthesis for affine fuzzy systems, which considers a constant bias term in each fuzzy rule [10–17]. These results considered the structural information in the rule base and introduced S-procedure to decrease the conservatism of the stability analysis. How-

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ever, it is required that a global Lyapunov function can be found for all LMIs, this constraint may leads to difficulties since such a Lyapunov matrix might not be found in many cases, especially for highly nonlinear complex systems. In [18], the authors proposed a stability result of fuzzy systems by using a piecewise quadratic Lyapunov function, which is a much richer class of Lyapunov function candidates and may lead to less conservatism than the global Lyapunov function. During the last few years, motivated by this result, a number of new controller design methods were proposed based on the piecewise continuous Lyapunov function [19–22]. Due to the constant bias term and the introduced S-procedure, most existing results on controller synthesis for affine fuzzy systems are obtained in the forms of bilinear matrix inequalities (BMIs). As we know, to deal with such non-convex problems, we need to design an iterative LMI algorithm and obtain an initial feasible solution, which is usually conservative and even impracticable. Decoupling the Lyapunov matrix and the system matrix by adding slack variables can effectively decrease the conservatism in multi-objective control problems. In [22], the authors considered the asynchronous output feedback control problem for affine fuzzy models using a piecewise Lyapunov function, by introducing slack variables, it shows that the synthesis conditions can be formulated in terms of LMIs which can be efficiently solved by interior-point methods.

It is worth noticing that all the mentioned controller design approaches for affine fuzzy systems are considered in the full frequency domain. However, in practice, sometimes, the frequency ranges of disturbances are known beforehand [23,24]. In particular, each design specification is often given not for the entire frequency range but for a certain frequency range of relevance. For instance, a closed-loop shaping control design typically requires small sensitivity in a low frequency range and small complementary sensitivity in a high frequency range. Thus a set of specifications would generally consist of different requirements in various frequency ranges. Fortunately, the Kalman–Yakubovich–Popov (KYP) lemma is generalized in [25] to characterize frequency domain inequalities with (semi)finite-frequency ranges in terms of linear matrix inequalities (LMIs). The generalized KYP lemma has been proven to be a powerful tool to treat frequency domain inequalities in finite frequency ranges [26–30]. However, due to the constant bias terms and introduced S-procedure of continuous-time affine fuzzy systems, the existing results in finite frequency are not suitable for such kinds of systems. To the best of our knowledge, there is no result about the control of nonlinear systems in the finite-frequency domain, which motivates us to make an effort in this paper.

The main objective of this paper is to derive a finite frequency performance analysis condition and propose a state feedback controller design method to meet the desired specifications in finite frequency ranges for nonlinear systems, which are described by affine fuzzy parts. First, motivated by Iwasaki et al. [30], a so-called finite-frequency H_∞ performance index is defined for the affine fuzzy system, which extends the standard H_∞ performance and contains the frequency information of disturbances. Then, by using a piecewise continuous Lyapunov function, a sufficient condition with the finite-frequency H_∞ performance is derived. Based on the obtained condition combined with some matrix decoupling techniques, piecewise controllers are designed in terms of linear matrix inequalities (LMIs) to deal with disturbances in low-, middle-, and high-frequency domain, respectively. Compared with the existing results, the proposed piecewise controller in finite frequency herein can get a better disturbance-attenuation performance when frequency ranges of disturbances are known in advance, and can deal with a larger class of fuzzy dynamic systems. Finally, two examples are given to illustrate the effectiveness of the proposed method.

This paper is organized as follows: following the introduction, the system description and the problem under consideration are given in Section 2. In Section 3, lemmas which are used throughout are proposed. An H_∞ controller design condition in the finite frequency is presented in Section 4. Numerical examples are given in Section 5 to show the effectiveness and superiority of the proposed method. Finally, conclusions are drawn in Section 6.

Notations: For a symmetric matrix M, $M > 0$ ($M < 0$) means that it is positive definite (negative definite), and M^T, M^*, $M^\perp$ denote its transpose, complex-conjugate transpose, and orthogonal complement, respectively. For a square matrix A, its Hermitian part is defined by $He(A) := (A + A^*)/2$. The symbol $\star$ is used in some matrix expressions to denote the transposed elements in the symmetric positions of a matrix.

2. System description and problem statement

2.1. System description

The following continuous-time affine fuzzy model can be used to represent a complex nonlinear system with both fuzzy inference rules and local analytic models as follows