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Group decision making based on multiplicative consistent reciprocal preference relations

Huimin Zhang

School of Management, Henan University of Technology, Zhengzhou, 450001, China

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Abstract

The main aim of this paper is to investigate the consistency and consensus of multiplicative consistent reciprocal preference relations. Based on the multiplicative consistency property, a straightforward method to obtain the priority vector for multiplicative consistent reciprocal preference relation is introduced together with some of its desired properties. In addition, some interesting properties on consistency and consensus for reciprocal preference relations are further investigated. A consensus optimization model for group decision making with reciprocal preference relations is proposed, which enables the decision makers to reach consensus with the highest overall consensus level. Numerical examples are provided to make a discussion and comparison with some existing similar methods.

Keywords: Multiplicative consistent reciprocal preference relation; Priority vector; Consistency; Consensus; Group decision making

1. Introduction

Group decision making (GDM) based on reciprocal preference relations (To avoid misunderstanding, in this paper, the elements of reciprocal preference relations are supposed to take the form of crisp numbers instead of fuzzy numbers) has received much research attentions over the past decades [2,8–10,19,21,22,25,26,34,39,50,55]. In the process of GDM, consistency and consensus are two very important issues which cause wide public concern of decision makers (DMs). Consistency measures the level of agreement among the preference values provided by the individual DMs, and consensus measures the level of agreement among the DMs on the solution of the problem [48].

The lack of consistency in decision making with preference relations is inclined to result in inconsistent conclusions. Many methods on consistency measure and improvement of preference relations with different forms have been presented successively. Xu and Wei [56] proposed an algorithm to improve the consistency of a multiplicative preference relation and gave an algorithm to derive a positive reciprocal matrix with acceptable consistency. Herrera-Viedma et al. [23] introduced a characterization of the consistency property defined by the additive transitivity property of the reciprocal preference relations and proposed a method for constructing consistent reciprocal preference relations. Ma

E-mail address: zhm76@126.com.

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et al. [32] introduced a method to identify the inconsistency and weak transitivity of a reciprocal preference relation and to improve its inconsistency to reach weak transitivity based on additive consistency. Dong et al. [17] presented a consistency index of linguistic preference relations and developed a consistency measure method for linguistic preference relations. Chiclana et al. [11] put forward a functional equation to model the "cardinal consistency in the strength of preferences" of reciprocal preference relations and concluded that multiplicative transitivity is the most appropriate property for modeling cardinal consistency of reciprocal preference relations. Ergu et al. [20] developed a method to measure the consistency for multiplicative preference relations. Siraj et al. [38] proposed an algorithm to improve ordinal consistency by identifying and eliminating intransitivities in multiplicative preference relations. Xia et al. [48] investigated the consistency of reciprocal preference relations based on the multiplicative consistency property and developed an algorithm to improve the consistency level of reciprocal preference relations. Liu et al. [29] proposed a definition of consistent triangular fuzzy reciprocal preference relations and studied several properties of consistent triangular fuzzy reciprocal preference relations. Wu and Chiclana [43] presented a novel consensus model for GDM problems with incomplete intuitionistic reciprocal preference relations and applied multiplicative consistency to estimate missing values. In order to unify different approaches on consistency of preference relations with different forms, Cavallo and D'Apuzzo [5] introduced pairwise comparison matrix over an Abelian linearly ordered group. In addition, Cavallo and D'Apuzzo [6] analyzed a condition of transitivity for a reciprocal matrix over an Abelian linearly ordered group and proposed corresponding tools for checking the transitivity and reliability of a weighting vector. Xia and Chen [49] proposed a consistency index is defined by constructing the nearest consistent pairwise comparison matrix on Abelian linearly ordered group from an inconsistent one, together with two consistency improving methods.

To help the DMs to reach consensus in GDM, many consensus models on preference relations with different forms have been developed. Xia et al. [48] investigated the consensus of reciprocal preference relations based on the multiplicative consistency property and proposed a consensus improving algorithms for individual reciprocal preference relations. Xu [53] presented some deviation measures to evaluate the consensus between the individual and the collective multiplicative preference relations and developed a consensus reaching process of GDM with incomplete multiplicative preference relations. Wu and Xu [46] provided a decision support model to aid the group consensus process while keeping an acceptable individual consistency for each DM. Palomares et al. [35] proposed a consensus model which incorporates the management of the group's attitude towards consensus by means of an extension of Ordered Weighted Averaging aggregation operators. Mata et al. [30] proposed an adaptive consensus support system model which is confined to multigranular linguistic contexts for GDM. Cabrerizo et al. [4] presented a consensus model to help experts in all phases of the consensus reaching process in GDM problems in an unbalanced fuzzy linguistic context with incomplete information. Herrera-Viedma et al. [24] gave a consensus model for GDM problems with different preference structures, preference orderings, utility values, reciprocal preference relations, and multiplicative preference relations. Chiclana et al. [7] developed a consensus model for GDM problems which proceeds from consistency to consensus. Wu and Chiclana [45] investigated a social network analysis trust-consensus based GDM model with interval-valued reciprocal preference relations, where the importance degrees of experts are determined by trust degree and consensus level. Chiclana et al. [12] presented a comparative study of the effect of the application of some different distance functions for measuring consensus in GDM. Mata et al. [31] proposed a consensus methodology which can directly manage linguistic term sets with different cardinality and semantic without the need to perform any transformation to unify the information. Wu and Chiclana [44] proposed a visual information feedback mechanism for GDM problems with triangular fuzzy complementary preference relations to identify experts, alternatives and corresponding preference values that contribute less to consensus. To provide a general framework for existing methods, Xia and Chen [49] defined a consensus index of individual pairwise comparison matrices and developed two consensus improving methods by introducing a general aggregation operator based on Abelian linearly ordered group.

This paper focuses on the consistency and consensus of multiplicative consistent reciprocal preference relations. To do this, the rest of this paper is structured as follows. In Section 2, a brief introduction to the basic notions is provided. Section 3 proposes a straightforward method to obtain the priority vector for a multiplicative consistent reciprocal preference relation and discusses some of its properties. In Section 4, some properties on consistency and consensus for reciprocal preference relations are further investigated. Section 5 proposes a consensus optimization model for GDM with reciprocal preference relations. In Section 6, the discussion and comparison with other similar methods are provided. Section 7 gives the conclusions.