Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Sets and Systems 282 (2016) 131–142
FUZZY sets and systems
www.elsevier.com/locate/fss

Common fixed point theorems in fuzzy metric spaces using the CLRg property

Antonio-Francisco Roldán-López-de-Hierro [a], Wutiphol Sintunavarat [b,*]

[a] Department of Mathematics, University of Jaén, Campus las Lagunillas s/n, 23071 Jaén, Spain
[b] Department of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University Rangsit Center, Pathumthani 12121, Thailand

Received 18 January 2014; received in revised form 4 August 2014; accepted 5 November 2014

Available online 12 November 2014

Abstract

The main aim of this paper is to obtain some new common fixed point theorems for weakly compatible mappings in fuzzy metric spaces (in the sense of Kramosil and Michálek) satisfying new contractive conditions employing the common limit in the range property. Our results generalize and extend some recent results contained in Jain et al. (2012) [12] to generalized contractive conditions under some control functions. In the process, some results about multidimensional common fixed points as coupled/tripled/quadrupled common fixed point results are derived from our main results.

Keywords: Fuzzy metric space; Coincidence point; Fixed point; Common limit in the range property; Fuzzy topology; Hadžić type t-norm

1. Introduction

The fixed point theory has applications in many areas such as in optimization theory, control theory, economics and many branches in analysis. Due to this fact, a number of authors have focused on the topic and have published some interesting fixed point theorems in this frame. In 1982, Sessa [24] first studied common fixed points results for weakly commuting pair of mappings. Later on, Jungck [13] generalized the idea of weakly commuting pair of mappings by introducing the notion of compatible mappings and showed that compatible pair of mappings commute on the set of coincidence points of the involved mappings. In 1996, Jungck [14] introduced the notion of weakly compatible mappings. Afterward, Aamri and Moutawakil [2] introduced the notion of property (E.A.) which is a special case of tangential property due to Sastry and Murthy [22]. In 2011, Sintunavarat and Kumam [25] obtained that the notions of property (E.A.) always requires the completeness (or closedness) of underlying subspaces for the existence of common fixed point. Hence they coined the idea of *commom limit in the range property* (called *CLR*) which relaxes the requirement of completeness (or closedness) of the underlying subspace. They also proved fixed point results via

* Corresponding author.
E-mail address: wutiphol@mathstat.sci.tu.ac.th (W. Sintunavarat).

http://dx.doi.org/10.1016/j.fss.2014.11.005
0165-0114/

this concept in fuzzy metric spaces. Most recently, Jain et al. [12] extended the concept of (CLR) property in the coupled case and also established coupled fixed point theorems for a pair of weakly compatible mapping along with (CLR) property in fuzzy metric spaces.

The aim of this paper is to study some new common fixed point theorems for weakly compatible mappings satisfying generalized contractive conditions under some control functions by using the common limit in the range property in fuzzy metric spaces. Under this condition, it is not necessary to assume the completeness of the space, which is an important advantage compared with the most of theorems in fixed point theory. Main results in this work generalize, unify and extend some recent results of Jain et al. [12] and many results in literature. As an extension of our main result, we state some results of multidimensional common fixed point as a coupled/tripled/quadruple common fixed point results. We illustrate our main result with an example.

2. Preliminaries

Some of the following preliminaries can also be found in [21]. Let N be a positive integer. Henceforth, X will denote a non-empty set and X^N will denote the product space $X \times X \times \overset{N}{\cdots} \times X$. Throughout this manuscript, n and m will denote positive integers, t will be a positive real number and $i, j \in \{1, 2, \ldots, N\}$. Let denote $\mathbb{I} = [0, 1]$. In the sequel, let $F : X^N \to X$ and $f, g : X \to X$ be three mappings. For brevity, $g(x)$ will be denoted by gx.

The main aim of the present paper is to guarantee existence and uniqueness of the following class of points.

Definition 1. Given $f, g : X \to X$, we will say that a point $x \in X$ is a:

- *fixed point of* f if $fx = x$;
- *coincidence point of* f *and* g if $fx = gx$;
- *common fixed point of* f *and* g if $fx = gx = x$.

Following Gnana-Bhaskar and Lakshmikantham (see [4]), given $F : X^2 \to X$ and $g : X \to X$, we will say that a point $(x, y) \in X^2$ is a

- *coupled fixed point of* F if $F(x, y) = x$ and $F(y, x) = y$;
- *coupled coincidence point of* F *and* g if $F(x, y) = gx$ and $F(y, x) = gy$;
- *coupled common fixed point of* F *and* g if $F(x, y) = gx = x$ and $F(y, x) = gy = y$.

Following Berinde and Borcut (see [3,5]), given $F : X^3 \to X$ and $g : X \to X$, we will say that a point $(x, y, z) \in X^3$ is a

- *tripled fixed point of* F if $F(x, y, z) = x$, $F(y, x, y) = y$ and $F(z, y, x) = z$;
- *tripled coincidence point of* F *and* g if $F(x, y, z) = gx$, $F(y, x, y) = gy$ and $F(z, y, x) = gz$;
- *tripled common fixed point of* F *and* g if $F(x, y, z) = gx = x$, $F(y, x, y) = gy = y$ and $F(z, y, x) = gz = z$.

Following Karapınar and Luong (see [15,16]), given $F : X^4 \to X$ and $g : X \to X$, we will say that a point $(x, y, z, t) \in X^4$ is a

- *quadrupled fixed point of* F if $F(x, y, z, t) = x$, $F(y, z, t, x) = y$, $F(z, t, x, y) = z$ and $F(t, x, y, z) = t$;
- *quadrupled coincidence point of* F *and* g if $F(x, y, z, t) = gx$, $F(y, z, t, x) = gy$, $F(z, t, x, y) = gz$ and $F(t, x, y, z) = gt$;
- *quadrupled common fixed point of* F *and* g if $F(x, y, z, t) = gx = x$, $F(y, z, t, x) = gy = y$, $F(z, t, x, y) = gz = z$ and $F(t, x, y, z) = gt = t$.

A notion of *multidimensional fixed/coincidence/commom point* was given in Roldán et al. [19,21]. In order to guarantee existence and uniqueness of the previous kind of points, we will use the following properties and notations.