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Fuzzy-based linguistic patterns as a tool for the flexible assessment of a priority vector obtained by pairwise comparisons

Jerzy Grobelny

Faculty of Computer Science and Management, Wrocław University of Technology, Wrocław, Poland

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Abstract

A novel approach to the assessment of a hierarchy vector generated by an expert pairwise comparison matrix is proposed. The 'quality' of a given hierarchy is measured using so-called 'linguistic patterns', i.e., specially constructed logical expressions, and an expert assessment matrix. The idea of this approach is based on a concept proposed previously as a flexible criterion for the domain of facility layout problems. The adaptation of the linguistic pattern approach to the assessment of a hierarchy vector also (by analogy to the analytic hierarchy process or AHP) facilitates evaluations of expert consistency. The proposed idea is shown to be less sensitive to potential expert errors than both the classical and fuzzy AHP techniques. The proposed approach is described in detail and illustrated with simple examples. Some features of the proposed approach were studied using computer simulation experiments and the results are reported together with the conclusions obtained.

Keywords: Analytic hierarchy process; Facility layout problem; Fuzzy set theory; Linear ordering; Linguistic variable; Multivalued logic; Possibility theory

1. Introduction

Pairwise comparisons are used widely to elicit personal preferences. The research results obtained using pairwise comparisons are typically represented as a square matrix. This notation is also used to obtain a hierarchy of preferences. In general, it is also possible to determine whether the pairwise evaluations are consistent, which means that the following transitivity relationship is preserved: if object A is preferred over B, and B over C, then A should also be preferred over C.

In the 19th century [31], researchers started using pairwise comparisons for subjective assessments when two physical stimuli differed in terms of intensity. Thus, various types of stimuli can be compared in pairs and with different ranges of intensities, thereby obtaining the following simple relationship: $dI/I = K = \text{const}$. The so-called "just-noticeable difference" depends on the magnitude of the stimulus compared I, which was formalized as the

E-mail address: jerzy.grobelny@pwr.edu.pl.
URL: http://JerzyGrobelny.com.

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Webber–Fechner law to describe the relationship between a psychological sensation (S) evoked by the physical intensity of a stimulus (I): $S = K \log(I)$, where K is a constant that depends on the character of the stimulus [31]. The scientific interest in determining the hierarchy of objects based on pairwise comparisons continued in the 1920s with the development of empirical psychology. The most important research in this field was initiated by Thurstone [39]. His model known as the "Law of Comparative Judgment" was based on investigations of the possibility of measuring purely subjective properties such as the strength of an individual's opinions or feelings about various matters and the subjectively perceived quality of things. This trend in empirical psychology was an extension of the aforementioned studies of the perception of physical stimuli, which can be measured objectively. A further development of the "Law of Comparative Judgment" concept led to the construction of the currently employed version, which is the logistic model referred to as Thurstone's Comparative Judgment (CJ) model. The essence of this extension is based on an assumption that the difference between the values of hidden objects is determined by the dominance of one object compared with another, which affects a person in a logarithmic manner, similar to the Webber–Fechner law. A specific area where the CJ model is applied is educational assessment. A modified version of the CJ approach was described by Pollit [29] and Scher [35].

The Webber–Fechner law motivated the construction of the scales used in the Analytic Hierarchy Process (AHP) method developed by Saaty [32–34], where a numerical ratio scale was assigned to nine linguistic expressions and used to find a hierarchy vector with the eigenvector technique. This method also facilitates assessments of the consistency among experts. AHP belongs to a group of highly popular and comprehensive approaches that utilize pairwise comparisons. AHP is applied in fields where expert knowledge is used to make decisions based on multiple criteria. For example, an extensive review of applications to marketing was provided previously [8,11]. Another study [37] presented a review of industrial applications of AHP based on 290 articles published between 1990 and 2009. This analysis showed that AHP has been applied mainly to various macro-problems (complex and real), as well as managerial and subjective-oriented problems.

Pairwise comparisons have also been used to develop methods that are not based directly on psychophysical research. For example, some employ the nonmetric multidimensional scaling technique proposed by Kruskal [21]. Psychophysical justifications of expert behavior are also ignored in the general linear ordering problem (LOP), which focuses mainly on procedures or algorithms for determining the optimal linear order vector based on pairwise comparisons, although there are no specific requirements about the scales applied and the methods used making judgments. The evaluation of the quality of the vector obtained is general and it does not refer to the ranks assigned to individual objects. This brief survey of the historical and applied aspects of searching hierarchies based on pairwise comparisons demonstrates that a great variety of techniques are employed and there are many applications areas for this approach.

It appears that the popularity of this approach is related to the simplicity of comparing only two alternatives at a time, as well as its empirically supported effectiveness. For example, Koczkodaj [19] demonstrated the substantial advantages of pairwise comparisons for the valid direct ranking of investigated objects.

This brief literature review shows that two main types of method dominate the modeling of hierarchy searching problems based on pairwise comparisons. The first type is the classical LOP, where the quality criterion has a holistic nature. The hierarchy is assessed as a whole and the only concern is the relative positioning of the objects. The second type comprises methods that rank objects based on the means of numbers defined on scales. The most popular of these methods is the AHP and its modifications. The main objective when establishing a hierarchy is exactly the same according to both approaches, but they employ different measurement scales for their comparisons and optimality criteria; thus, the methods used to search for optimal hierarchies are also different.

1.1. Classic LOP

In a classical LOP, the measurement scale is not defined in detail and it is assumed that it can be any ordinal scale. The optimal hierarchy vector in this model is a permutation (p) of objects such that the preference matrix (A) sorted according to p has a maximal sum of assessment values located above the diagonal.

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \mathrm{A}_{p^*(i)p^*(j)} \to \max \tag{1}$$

The interpretation of this criterion is particularly obvious for zero-one comparisons, i.e., if object i is preferred over object j, the value one appears in the ij cell of the comparisons matrix, otherwise the ij cell contains zero. If this