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Branch-and-price algorithm for fuzzy integer programming problems with block angular structure ☆

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Abstract

This paper deals with fuzzy integer linear programming problems with block angular structure in which the fuzzy constraints are simplified by using possibility and necessity relations. This main fuzzy problem is efficiently decomposed and is solved by a branch-and-price algorithm. In the nodes of the branch-and-price tree, the linear relaxation of the problem is solved by applying a column generation method. Also, the relationship between the optimal solutions of this problem under possibility and necessity relations is derived. To show the validation of the proposed algorithm, some results are proved. In addition, the application of this algorithm is illustrated on fuzzy multicommodity flow problem. For this case, a new branching scheme is proposed to preserve the network structure of the subproblems which are produced in the column generation method. Some examples are solved and their results are compared with the previous works. Also, the results of the proposed algorithm are reported on some large-scale benchmark instances.

Keywords: Fuzzy optimization; Branch-and-price algorithm; Branching rule; Column generation; Integer multicommodity flow problem; Possibility and necessity relations

1. Introduction

Branch-and-price algorithm was introduced in the late 1990s to obtain the optimal solution of the mixed integer programming problems [3,6]. This algorithm combines column generation method with the branch-and-bound algorithm. Column generation method decomposes the problem into the smaller subproblems [11,15]. For some references, it can be refereed to [7,22,28,30]. However, decomposition algorithms are hardly ever applied to solve fuzzy programming problems. Fuzzy goal programming problem with block angular structure has been studied in [29] in which the

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problem has been divided into several independent linear subproblems under some conditions. In continuation of this work, we consider fuzzy integer programming problem with block angular structure with fuzzy objective function as well as fuzzy constraints. Fuzzy constrains are simplified by applying possibility and necessity relations. The concept of (α, β)-optimal solution is introduced. The problem is efficiently decomposed to provide an opportunity to solve this complex problem with the branch-and-price algorithm. Note that, the important key for developing the decomposition algorithm is the representation theorem which is extended in the current paper.

As an important application of the introduced problem, fuzzy multicommodity flow problem is considered. Multicommodity flow problem was initially proposed by Ford and Fulkerson [13]. Ghatee and Hashemi [14] first introduced fuzzy concepts in multicommodity flow problem. Also, Ciappina et al. ([8] and [9]) applied Dantzig–Wolfe decomposition method to solve this problem with fuzzy cost. However, fuzzy demands and capacities were not considered. In this paper, uncertainty is considered in the cost coefficients, as well as the demand and available capacity parameters in the multicommodity flow problem. In addition, the integral restriction on the variables is considered in an integer programming problem. The branch-and-price algorithm [2,5,12] for this problem is extended in the current paper. Also, a new branching scheme is proposed to transform the subproblems into simple shortest path problems. Thus, the linear relaxation of the problem can be efficiently solved in the nodes of branch-and-price tree.

The rest of the paper is organized as follows: in the next section, basic definitions of fuzzy sets, fuzzy quantities and fuzzy possibility and necessity relations are reviewed. Then, fuzzy integer programming problem with block angular structure is introduced. In Section 3, firstly the decomposition of the problem is illustrated. Branch-and-price algorithm under possibility and necessity relations is developed in Sections 3.1 and 3.2, respectively. Then, the relationships between the feasible solutions sets and the optimal solutions of the problem under possibility and necessity relations are investigated in Section 3.3. The application of the proposed approach to solve fuzzy integer multicommodity network flow problem is explained in Section 4. Some numerical examples are given as well. The final section provides a brief conclusion and future directions.

2. Preliminaries

2.1. Possibility and necessity relations

The structure of this section is adopted from [16,17,26]. Let $\widetilde{a}$ be a fuzzy subset of X with the membership function $\mu_{\widetilde{a}} : X \to [0, 1]$. The α-cut of $\widetilde{a}$ is denoted with $[\widetilde{a}]_\alpha$ and it is defined as follows:

$$[\widetilde{a}]_\alpha = \begin{cases} \{x \in X | \mu_{\widetilde{a}}(x) \geq \alpha\} & \alpha \in (0, 1], \\ cl\{x \in X | \mu_{\widetilde{a}}(x) > 0\} & \alpha = 0, \end{cases}$$

where clB means the closure of the set B. $[\widetilde{a}]_0$ is usually called the support of $\widetilde{a}$. A fuzzy subset $\widetilde{a}$ of X is called closed, bounded, compact or convex, if for every $\alpha \in [0, 1]$, $[\widetilde{a}]_\alpha$ is closed, bounded, compact or convex, respectively. Moreover, $\widetilde{a}$ is said to be normal, if $[\widetilde{a}]_1$ is not empty [31].

Definition 2.1. (See [26].) A fuzzy subset $\widetilde{a}$ of $\mathbb{R}$ is called a fuzzy quantity, if $\widetilde{a}$ is a normal, compact fuzzy subset. Moreover, there exists $a, b, c, d \in \mathbb{R}$, $-\infty \leq a \leq b \leq c \leq d \leq +\infty$ such that

$\mu_{\widetilde{a}}(t) = 0$ if $t < a$ or $t > d$,
$\mu_{\widetilde{a}}(t)$ is strictly increasing if $a \leq t \leq b$,
$\mu_{\widetilde{a}}(t) = 1$ if $b \leq t \leq c$,
$\mu_{\widetilde{a}}(t)$ is strictly decreasing if $c \leq t \leq d$.

In order to compare fuzzy quantities $\widetilde{a}$ and $\widetilde{b}$, the concept of possibility and necessity relations is defined as the following:

Definition 2.2. (See [24,26].) Let $\widetilde{a}$ and $\widetilde{b}$ be two fuzzy quantities with the membership functions $\mu_{\widetilde{a}}$ and $\mu_{\widetilde{b}}$, respectively. The possibility relation and the necessity relation can be defined as the following:

$\Pi(\widetilde{a}, \widetilde{b}) = \sup_{x \leq y}\{\min(\mu_{\widetilde{a}}(x), \mu_{\widetilde{b}}(y))\}$,