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Fuzzy decision making for multiobjective stochastic programming problems

H. Yano [*]

School of Humanities and Social Sciences, Nagoya City University, 467-8501 Nagoya, Japan

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Abstract

In this study, we propose a fuzzy decision making method for multiobjective stochastic linear programming problems with variance covariance matrices (MOSLPs), in which the criteria of probability maximization and fractile optimization are simultaneously considered. For a probability maximization model for an MOSLP, the decision maker is required to specify permissible objective levels. However, fewer values of permissible objective levels for minimization problems result in fewer values of the corresponding distribution function because of the conflicts between them. Similarly, for a fractile optimization model for an MOSLP, the decision maker is required to specify permissible probability levels. However, due to conflicts between the probability levels and the corresponding objective functions, larger values of permissible probability levels result in larger values of the corresponding objective function for minimization problems. In this study, it is assumed that the decision maker has fuzzy goals not only for permissible objective levels of a probability maximization model but also for permissible probability levels of a fractile optimization model, and such fuzzy goals are quantified by eliciting the corresponding membership functions. On the basis of the fuzzy decision, the satisfactory solution of the decision maker is obtained by applying a convex programming technique.

Keywords: Multiobjective stochastic linear programming; Variance covariance matrices; Probability maximization; Fractile optimization; Fuzzy decision

1. Introduction

When an expert attempts to formulate a mathematical programming problem that reflects a real-world decision making situation, they often encounter difficult problems such as estimating the parameters of the objective function and the constraints. In traditional approaches, these parameters are fixed as constants in a subjective manner, and well-known linear or nonlinear programming problems and/or multiobjective programming problems are formulated. However, consider a crop planning problem at the farm level [6,7,10]. In a crop planning problem, a farmer or an agricultural manager wants to earn as much money as possible by effectively utilizing their farmland. Suppose that

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[*] Correspondence to: 1 Yamanohata, Mizuho-cho, Mizuho-ku, Nagoya 467-8501, Japan. Tel.: +81 52 872 5182; fax: +81 52 872 1531.
E-mail address: yano@hum.nagoya-cu.ac.jp.

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each decision variable represents the cultivation area for each of several types of crops and that parameters of the objective function (total profit) represent the profit coefficients by unit area for each crop. Unfortunately, in an actual agricultural decision problem, crop yield upon harvesting often depends on not only uncertain weather conditions but also on the immeasurable effects of nature. Therefore, agricultural profit coefficients are uncertain values at harvest time.

To deal with such uncertain decision situations, two types of approaches (stochastic programming and fuzzy programming) have been proposed. In stochastic programming approaches, two-stage programming approaches [4] and chance constrained programming models [2] have been investigated. These approaches were extended to multiobjective stochastic programming problems (MOSPs) by using a goal programming technique [3]. An interactive method called STRANGE was proposed for MOSPs with discrete random variables [18]. Caballero et al. [1] and Stancu-Minasian [16] defined the Pareto optimality concept for MOSPs on the basis of a probability maximization model and a fractile optimization model. An overview for solving MOSPs was described by Stancu-Minasian [17]. In fuzzy programming approaches, various types of fuzzy programming problems have been formulated and investigated [11,12,21]. As a natural extension, a multiobjective fuzzy programming technique was first proposed by Zimmermann [20], and many methods have since been proposed [13,21].

Recently, to simultaneously address probabilistic uncertainty and fuzzy one, the hybrid approaches of stochastic programming and fuzzy programming have been proposed [8]. In particular, Sakawa et al. [15] proposed an interactive fuzzy satisficing method for multiobjective linear programming problem with random variable coefficients. In their method, a probability maximization model was adopted to transform stochastic programming problems into well-defined mathematical programming ones. When adopting a probability maximization model, it is required that the decision maker specifies permissible levels for objective functions (called permissible objective levels) in their subjective manner in advance. However, it seems to be very difficult to specify such values in advance, as conflicts exist among permissible objective levels and their corresponding distribution function values. On the other hand, when a fractile optimization model is adopted [14], it is required that the decision maker specifies permissible levels for the probability levels (called permissible probability levels) in their subjective manner in advance. Similar to a probability maximization model, conflicts exist among permissible probability levels and the corresponding objective function values. In order to deal with such difficulties, Inuiguchi et al. [9] have already proposed a flexible programming model for single objective stochastic programming problems, in which a permissible objective level is set automatically through the fuzzy decision. As a natural extension of Inuiguchi and Sakawa [9], we previously proposed an interactive method for the specialized formulation of multiobjective stochastic programming problems to obtain a satisfactory solution, in which it is assumed that any coefficients in the objective function are expressed by only one random variable [19].

In this study, we propose a fuzzy decision making method to obtain a satisfactory solution for the generalized formulation of multiobjective stochastic linear programming problems with variance covariance matrices, in which the proper balance between permissible objective levels and permissible probability levels is attained through the fuzzy decision [13,21]. To discuss a probability maximization model and a fractile optimization model for an MOSLP, the P-Pareto optimal solution concept and the F-Pareto optimal solution concept are defined in Section 2. In Section 3, a fuzzy decision making method based on a probability maximization model is proposed for an MOSLP. In Section 4, a fuzzy decision making method based on a fractile optimization model is proposed for an MOSLP. An illustrative numerical example of an MOSLP demonstrates the efficiency of the proposed method in Section 5, through comparison with the previous methods.

2. MOSLP through a probability maximization model and a fractile optimization model

In this section, we focus on the MOSLP, which is formally formulated as follows.

[MOSLP]

$$\min \bar{\boldsymbol{C}}\boldsymbol{x} = (\bar{\boldsymbol{c}}_1\boldsymbol{x}, \cdots, \bar{\boldsymbol{c}}_k\boldsymbol{x}) \tag{1}$$

subject to

$$A\boldsymbol{x} \le \bar{\boldsymbol{b}}, \ \boldsymbol{x} \ge \boldsymbol{0} \tag{2}$$