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A comprehensive study of implicator–conjunctor-based and noise-tolerant fuzzy rough sets: Definitions, properties and robustness analysis

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Abstract

Both rough and fuzzy set theories offer interesting tools for dealing with imperfect data: while the former allows us to work with uncertain and incomplete information, the latter provides a formal setting for vague concepts. The two theories are highly compatible, and since the late 1980s many researchers have studied their hybridization. In this paper, we critically evaluate most relevant fuzzy rough set models proposed in the literature. To this end, we establish a formally correct and unified mathematical framework for them. Both implicator–conjunctor-based definitions and noise-tolerant models are studied. We evaluate these models on two different fronts: firstly, we discuss which properties of the original rough set model can be maintained and secondly, we examine how robust they are against both class and attribute noise. By highlighting the benefits and drawbacks of the different fuzzy rough set models, this study appears a necessary first step to propose and develop new models in future research.

Keywords: Fuzzy sets; Rough sets; Hybridization; Lower and upper approximation; Noise-tolerance

1. Introduction

Rough set theory was originally proposed by Pawlak [47] in 1982 to deal with uncertainty due to incompleteness and indiscernibility. The basic idea of rough set theory is that it provides a lower and upper approximation of a concept with respect to a binary indiscernibility relation. The lower approximation contains all the elements of the universe certainly belonging to the concept, while the upper approximation contains the elements possibly belonging to the

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concept. In the original model of Pawlak, an equivalence relation is used to model indiscernibility. Yet, many authors have generalized Pawlak's model by using binary non-equivalence relations (see e.g. [48,49] for a survey).

Applications of rough set theory are widespread and are especially prominent in data analysis [33,35] and more specific in feature selection and classification [53]. However, since the traditional rough set is designed to process qualitative (discrete) data, it faces important limitations when dealing with real-valued data sets [31]. Fuzzy set theory proposed in 1965 by Zadeh [68] is very useful to overcome these limitations, as it can deal effectively with vague concepts and graded indiscernibility.

It was recognized early that both theories are complementary, rather than competitive. To that end, rough set theory has been extended in two ways [14]. Rough fuzzy set theory discusses the approximation of a fuzzy set by a crisp relation. If moreover the indiscernibility relation to distinguish different objects is fuzzy as well, fuzzy rough set theory is considered. Since every crisp relation can be seen as a special case of a fuzzy relation, all results obtained in fuzzy rough set theory also hold for rough fuzzy set theory.

The vestiges of fuzzy rough set theory date back to the late 1980s, and originate from work by Fariñas del Cerro and Prade [12], Dubois and Prade [13], Nakamura [45] and Wygralak [62]. From 1990 onwards, research on the hybridization between rough sets and fuzzy sets flourished. The inspiration to combine rough and fuzzy set theory was found in different mathematical fields. For instance, Lin [34] studied fuzzy rough sets using generalized topological spaces (Frechet spaces) and Nanda and Majumdar [46] discussed fuzzy rough sets based on an algebraic approach. Moreover, Thiele [54] examined the relationship with fuzzy modal logic. Later on, Yao [66] and Liu [39] used level sets to combine fuzzy and rough set theory.

This work focuses on fuzzy rough set models using fuzzy relations and fuzzy logical connectives. The seminal papers of Dubois and Prade [14,15] are probably the most important in the evolution of these fuzzy rough set models, since they influenced numerous authors who used different fuzzy logical connectives and fuzzy relations. Essential work was done by Morsi and Yakout [44] who studied both constructive and axiomatic approaches and by Radzikowska and Kerre [51] who defined fuzzy rough sets based on three general classes of fuzzy implicators: S-, R- and QL-implicators. However, despite generalizing the fuzzy connectives, they still used fuzzy similarity relations. A first attempt to use reflexive fuzzy relations instead of fuzzy similarity relations was done by Greco et al. [22,23]. Thereafter, Wu et al. [60,61] were the first to consider general fuzzy relations. Besides generalizing the fuzzy relation, Mi et al. [40,41] considered conjunctors instead of t-norms. Furthermore, Yeung et al. [67] discussed two pairs of dual approximation operators from both a constructive and an axiomatic point of view. Hu et al. [26,28] for their part studied fuzzy relations based on kernel functions. In this work, we consider all these different proposals within a general Implicator–Conjunctor (IC) based fuzzy rough set model that encapsulates all of them, as discussed in Section 3.1.

However, the aforementioned models only consider the worst and best performing objects to determine the fuzzy rough lower and upper approximations respectively. Consequently, these approximations are sensitive to noisy and/or outlying samples. This, in turn, impacts the robustness of data analysis applications based on them, such as attribute selection and classification. To mitigate this problem in the crisp case, Ziarko [70] proposed the Variable Precision Rough Set (VPRS) model in 1993. This model also served as a starting point for the design of several noise-tolerant fuzzy rough set approaches, such as [5,7,17,18,24,25,42,43,65,69], which will be discussed in detail in Section 4.

In this paper, we critically evaluate most relevant fuzzy rough set models proposed in the literature. To this end, we establish a formally correct and unified mathematical framework for them. A structured and critical analysis of the current research of constructive methods for fuzzy rough set models is presented. Note that we do not consider axiomatic approaches (see e.g. [36,37,41,44,50,59–61,67]). We review the definitions of noise-tolerant models, generalizing them where appropriate and in some cases applying modifications to correct errors in the original proposal. Where applicable, we also establish relationships between these models and the corresponding IC based definitions, as well as Pawlak's and Ziarko's crisp approaches. Furthermore, we examine which theoretical properties of traditional rough sets and IC fuzzy rough sets can still be maintained for the noise-tolerant models; indeed, similarly as for Ziarko's VPRS model, providing mechanisms for making the approximations less strict usually involves sacrificing some desirable properties. Finally, we evaluate whether the considered approaches really live up to the claim of being more "robust" approximations, by performing a stability analysis on four real datasets, and comparing them to the IC model. This will allow us to obtain a comprehensive overview of the benefits and the drawbacks of the robust fuzzy rough set models, in order to acquire the expertise for future research opportunities.

The remainder of this article is structured as follows: in Section 2, we summarize preliminary definitions concerning fuzzy logical connectives, fuzzy sets and relations, and rough set theory. In Section 3, we introduce the general IC