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Granular variable precision fuzzy rough sets with general fuzzy relations

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Abstract

The variable precision (θ, σ)-fuzzy rough sets were proposed to remedy the defects of preexisting fuzzy rough set models. However, the variable precision (θ, σ)-fuzzy rough sets were only defined and investigated on fuzzy $*$-similarity relations. In this paper, the granular variable precision fuzzy rough sets with general fuzzy relations are proposed on arbitrary fuzzy relations. The equivalent expressions of the approximation operators are given with fuzzy (co)implications on arbitrary fuzzy relations, which can calculate efficiently the approximation operators. The granular variable precision fuzzy rough sets are characterized from the constructive approach, which are investigated on different fuzzy relations. The conclusions on the variable precision (θ, σ)-fuzzy rough sets are generalized into the granular variable precision fuzzy rough sets.

Keywords: Fuzzy rough sets; Fuzzy granules; Granular variable precision fuzzy rough sets; Fuzzy implications

1. Introduction

Pawlak proposed rough sets in 1982 to deal with uncertain, incomplete and vague information [24,25]. Rough sets theory has been successfully used in many areas [29,30] including pattern recognition, data mining and automated knowledge acquisition. However, Pawlak defined rough sets with equivalence relations and crisp sets, which are shown to be too strict to limit the applicability of Pawlak's rough set model. Moreover, rough sets are sensitive to misclassification and perturbation, which represents erroneous or missing classification [38].

In recent years, rough set theory is investigated by constructive and axiomatic approaches. To process fuzziness of concepts and vagueness of information in decision making, Dubois and Prade [12] proposed fuzzy rough sets, which were further investigated and expanded with different fuzzy logic operations and binary fuzzy relations in [5,19,20,23,28,31–33,36,39]. Moreover, some of them are studied with addition topological or algebraic structures

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[26,27]. Different fuzzy rough sets can deal with real-valued data sets. On the other hand, Ziarko [40] proposed variable precision rough sets to process erroneous or missing classification, which were also generalized in a fuzzy environment [8,14,17,21,22,37].

Chen et al. [6] combined granular computing [2,11] with fuzzy rough sets, which is a new approach to develop fuzzy rough sets. On the basis of fuzzy granules [6], the granular (θ, σ)-fuzzy rough sets [6] were proposed and investigated on fuzzy $*$-similarity relations, which are equivalent to fuzzy rough sets defined by the membership functions (see Theorem 4.1.3 in [6]). As it was concluded in [34], some of fuzzy rough sets [16,18,38] are still sensitive to mislabeled samples and others [8,14,17,21,22,37] have only considered relative errors. To remedy these defects, Yao et al. [34] proposed the variable precision (θ, σ)-fuzzy rough sets based on fuzzy granules when the fuzzy relations are fuzzy $*$-similarity relations, which improved the granular (θ, σ)-fuzzy rough sets. The fuzzy $*$-similarity relations are useful in attribute reduction based on variable precision (θ, σ)-fuzzy rough sets, but the fuzzy $*$-similarity relations appear too strict for the investigation of the variable precision (θ, σ)-fuzzy rough sets from the mathematical point of view. Moreover, there has been no comparative analysis of the properties that hold for the variable precision (θ, σ)-fuzzy rough sets when the fuzzy relations are arbitrary, which are called the granular variable precision fuzzy rough sets here. This suggests that the granular variable precision fuzzy rough sets should be studied more thoroughly in this context. Following on from this, we study the granular variable precision fuzzy rough sets from the theoretical perspective. Equivalent expressions of the approximation operators are given with fuzzy (co)implications on arbitrary fuzzy relations, which make sure to calculate the approximation operators efficiently. The lower rough approximation operator and upper rough approximation operator are comparable in rough sets, which has been considered to be a fundamental property in many published works [7,9,35]. Since granular variable precision fuzzy rough sets are proposed to remedy the defects of preexisting fuzzy rough set models, such as mislabeled samples and relative errors, granular variable precision fuzzy rough sets do not satisfy this fundamental property, which has been analyzed with examples. Moreover, the granular variable precision fuzzy rough sets are characterized on different fuzzy relations such as serial, reflexive and $*$-transitive ones. It is pointed out that all conclusions on the variable precision (θ, σ)-fuzzy rough sets in [34] still hold in the granular variable precision fuzzy rough sets when the fuzzy relations are fuzzy $*$-preorders, that is, symmetry of fuzzy relations does not contribute to further special properties of the variable precision (θ, σ)-fuzzy rough sets.

The content of the paper is organized as follows. In Section 2, we recall some fundamental concepts and related properties about fuzzy implications and fuzzy coimplications based on left-continuous t-norms and right-continuous t-conorms, respectively. In Section 3, we propose the granular variable precision fuzzy rough sets with general fuzzy relations and give equivalent expressions of the approximation operators. We discuss when the granular variable precision fuzzy rough sets satisfy the comparable property between lower rough approximation operator and upper rough approximation operator. Moreover, the granular variable precision fuzzy rough sets are investigated when the fuzzy relations and fuzzy sets degenerate into crisp relations and sets, respectively. Section 4 characterizes the granular variable precision fuzzy rough sets on different fuzzy relations. We list a table to show the differences and connections between the variable precision (θ, σ)-fuzzy rough sets and granular variable precision fuzzy rough sets. In the final section, we present some conclusions of our research.

2. Preliminaries

Let X be a finite set called universe, and $\mathscr{F}(X)$ be the family of all fuzzy sets on X. For all crisp sets A, $|A|$ denotes the cardinality of the set A. A fuzzy set $A \in \mathscr{F}(X)$ is a *constant*, if $A(x) = \alpha$ for all $x \in X$, denoted as α_X. Moreover, A is called a *fuzzy point*, if for all $x \in X$,

$$A(x) = \begin{cases} \alpha, & x = y; \\ 0, & x \neq y; \end{cases}$$

denoted as y_α. A function $N : [0, 1] \to [0, 1]$ is called an *involutive negation*, if it is decreasing and satisfies $N(N(a)) = a$ for all $a \in [0, 1]$. The involutive negation $N(a) = 1 - a$ for all $a \in [0, 1]$ is usually referred as the standard negation. Assume that $\triangle$ and $\triangledown$ are two binary operations on $[0, 1]$, then they are said to be *dual* with respect to (w.r.t., for short) N, if for all $a, b \in [0, 1]$, $N(a \triangle b) = N(a) \triangledown N(b)$.