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Similarity relations in fuzzy attribute-oriented concept lattices

Gabriel Ciobanu, Cristian Văideanu

Romanian Academy, Institute of Computer Science, and A.I. Cuza University of Iaşi, Faculty of Mathematics, Blvd Carol I no. 11, 700506 Iaşi, Romania

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Abstract

The study of concept lattices and oriented concept lattices with fuzzy attributes provides complementary conceptual structures, which can be used to search, analyze and extract information from large datasets. In this paper, we put forward new types of similarity relations between objects or attributes in fuzzy attribute-oriented concept lattices. We analyze how these similarities are related using the definitions and considering the reducibility between antitone and isotone fuzzy concept lattices as well. A comparison is made between two relations which measure the similarity of fuzzy oriented concepts. These relations are employed in factorizing the fuzzy attribute-oriented concept lattice, in order to reduce its complexity.

Keywords: Similarity relation; Formal concept analysis with fuzzy attributes; Fuzzy attribute-oriented concept lattice

1. Introduction

Formal concept analysis (FCA) is a mathematical theory, originally proposed by Rudolf Wille [1] in order to provide support for conceptual data analysis and knowledge discovery. FCA techniques allow to build a hierarchical structure of clusters from a dataset representing a set of objects together with their attributes (properties) and to use this conceptual hierarchy as a knowledge extraction and representation tool.

Given such a dataset called formal context, two important Galois connections can be defined between the powerset of objects and the powerset of attributes. Wille studied an antitone Galois connection and introduced the notions of formal concept and concept lattice. In the last decades several extensions of his model have been developed. Thus, *conceptual* scaling was studied as a technique for dealing with the so-called many-valued contexts, *pattern* structures were defined in [2] as analogues of contexts but with a semilattice of patterns instead of attributes, while *triadic* concept analysis was founded on the notion of triadic context [3].

Another method to define conceptual structures by addressing the objectives of knowledge processing was provided by the modal style operators framework. Based on two approximation operators, which form an isotone Galois connection, Gediga and Duntsch [4] defined a new type of concept lattice: the attribute-oriented concept lattice. Using

E-mail addresses: gabriel@info.uaic.ro (G. Ciobanu), cvaideanu@gmail.com (C. Văideanu).

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sets of objects instead of sets of attributes, Yao [5] introduced in a similar way the notion of object-oriented concept lattice.

Formal concept analysis with fuzzy attributes is a generalization of the classical model, considering the cases in which uncertain information occurs in the representation of the data. Belohlavek [6] and independently Pollandt [7] considered a residuated lattice, a formal fuzzy context, and defined a pair of fuzzy derivation operators. Based on this antitone Galois connection which is similar to that defined by Wille, they introduced the notions of formal fuzzy concept and fuzzy concept lattice.

Gediga and Duntsch's model was generalized to a fuzzy setting by Georgescu and Popescu (see [8,9]). Using a family of fuzzy isotone Galois connections, they developed a general method to form the context and to build the concepts. Particularly, the notions of attribute-oriented and object-oriented formal fuzzy concept can be obtained from their dynamical model. Other important approaches for the theory of FCA with fuzzy attributes can be also found in the literature (see [10] and [11]).

Both theories, of fuzzy concept lattices and fuzzy oriented concept lattices, provide related conceptual structures which can be used to explore, search and compare different data. Since for dense formal contexts the size of the corresponding concept lattices can be very large, the problem of generating the set of concepts is not an easy one. Many techniques have been developed to overcome this drawback, such as classification, pattern recognition or cluster analysis. An important step when comparing objects in clustering methods is achieved on the basis of a measure which determines how similar or dissimilar the concepts of a structure are. Various similarities which measure the degree of resemblance between formal concepts have been proposed in the recent years [6,12–15].

Belohlavek [12] studied different types of similarity relations between objects, attributes or concepts in the case of antitone conceptual structures with fuzzy attributes. Using a similarity measure between concepts, he provided a parameterized method to factorize the fuzzy concept lattice and to build the factor lattice. Belohlavek et al. [6] developed an algorithm which yields a faster way to compute the factor lattice. Thus instead of building the concept lattice first, and then the similarity blocks, they provided a method of obtaining the factor lattice directly from the input data.

In this paper we address the same issues as Belohlavek, but in the case of isotone fuzzy concept lattices. Thus, we propose new types of similarity relations for fuzzy attribute-oriented concept lattices (a similar approach can be developed for fuzzy object-oriented concept lattices). First, we use an isotone fuzzy Galois connection to introduce two similarities between objects, one induced by the fuzzy attribute-oriented concept lattice, and another induced directly by the fuzzy context. We prove that, unlike in the antitone fuzzy case where the two similarities are equal (see [12]), in our case they are only in a relationship of inequality. However, they can be equal under certain conditions; we find a sufficient condition for these similarities to be equal. Next, by a similar reasoning, we define two similarities between attributes and prove that they are equal, precisely as in Belohlavek's case. We also introduce two similarities between attribute-oriented fuzzy concepts, which are induced by their extents or intents, and we prove that they are equal.

It is well known that concept lattices of antitone and isotone Galois connections are isomorphic as lattices in the case of binary incidence relations. This duality means that once a result is proved in one structure, it results that an analogous statement automatically holds in the other structure. However, as Georgescu and Popescu [8] noticed, due to the lack of the double negation property, this duality is no longer true for "classical" formal fuzzy contexts. We understand here by "classical" fuzzy context a fuzzy setting in which the complement of the incidence fuzzy relation is defined by using the residuum with respect to 0. Consequently, although very similar, the structures associated to isotone Galois connections, which we define, are not dual to those introduced by Belohlavek in [12] for antitone Galois connections. Belohlavek and Konecny [16] introduced a new notion of complement for a fuzzy relation and proved that a part of the mutual reducibility between the antitone and isotone cases remains true in this new environment.

In our work we also analyze how the properties of the similarities we introduced can be deduced from the results in the antitone case by applying this new type of duality.

We find that for some statements this technique provides straightforward proofs, while in other situations the direct arguments we provide are better. We also show that not all similarities we introduce are preserved via the isomorphism between the antitone and isotone fuzzy concept lattices.

A further contribution of this work consists in defining a tolerance relation on the set of attribute-oriented fuzzy concepts based on the new types of similarity. This relation can be used to factorize the attribute-oriented concept