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Convexity of n-dimensional fuzzy number-valued functions and its applications ☆

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Abstract

The aim of this study is to generalize convexity from vector-valued maps to n-dimensional fuzzy number-valued functions and to investigate some relations among the convexity and generalized convexity of n-dimensional fuzzy number-valued functions, which are based on a new ordering of n-dimensional fuzzy numbers proposed in this study. Furthermore, some criteria are obtained for the convexity and generalized convexity under the conditions of lower and upper semicontinuity, respectively. Finally, we study the local–global minimum properties of the convex fuzzy number-valued functions.

Keywords: Convex fuzzy number-valued functions; Fuzzy numbers; Fuzzy optimization; n-Dimensional fuzzy number-valued functions; Semicontinuity

1. Introduction

Since the concept and operations of fuzzy sets were introduced by Zadeh [22], many researchers have been involved in the development of various aspects of the theory and applications of fuzzy sets. Subsequently, Zadeh proposed the notion of fuzzy numbers in [23–25] and fuzzy numbers have also been investigated extensively.

Convexity plays a key role in mathematical economics, engineering, management science, and optimization theory [14,15]. Therefore, research into convexity and generalized convexity is one of the most important aspects of mathematical programming. In 1992, Nanda and Kar [12] proposed the concept of convex fuzzy mapping and they established criteria for convex fuzzy mappings. Subsequently, many studies have addressed the development of various aspects of convex fuzzy mapping and its applications to fuzzy optimization [1,2,7,18,19]. Different types of convex fuzzy mappings and generalized convex fuzzy mappings have been proposed and studied with applications to fuzzy nonlinear programming.

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It is well known that convexity, generalized convexity, semicontinuity and their relationships have central roles in classical convex analysis and optimization [4,17]. The concept of the lower and upper semicontinuity of fuzzy mappings based on the Hausdorff separation was introduced by Diamond and Kloeden [6]. In 2006, Bao and Wu [3] introduced a new concept of lower and upper semicontinuity of fuzzy mappings based on the "fuzz-max" order of fuzzy numbers, and they obtained the criteria for convex fuzzy mappings under lower and upper semicontinuity conditions, respectively.

To the best of our knowledge, very few studies have investigated the convexity of n-dimensional fuzzy number-valued functions, and the present study is the first report to address this problem. Motivated by earlier research and by the importance of the concept of convexity, we introduce the concept of a convex fuzzy number-valued function and a generalized convex fuzzy number-valued function, which are based on a new ordering $\preceq_c$ of n-dimensional fuzzy numbers proposed in this study. In fuzzy optimization theory, we also need to consider the relationships among semicontinuity, convexity, and generalized convexity, which are significant. We obtain a necessary and sufficient condition that the lower (upper) semicontinuity implies convexity for an n-dimensional fuzzy number-valued function.

First, we provide the preliminary terminology used throughout this study and we introduce a new ordering $\preceq_c$ of n-dimensional fuzzy numbers, which has not been discussed previously. In Section 3, motivated by earlier research, we introduce a new type of n-dimensional fuzzy number-valued function called a convex fuzzy number-valued function based on the proposed ordering $\preceq_c$. We establish the relationships among convex, strictly convex, quasiconvex, and strictly quasiconvex fuzzy number-valued functions. In Section 4, we characterize the convexity and generalized convexity based on the semicontinuity. We obtain a necessary and sufficient condition that the lower (upper) semicontinuity implies convexity, and we prove that the convexity is equivalent to its weak convexity for a lower (and upper) semicontinuous n-dimensional fuzzy number-valued function. Finally, we give some results based on an application to convex fuzzy programming in Section 5.

2. Preliminaries

Throughout this study, R^n denotes the n-dimensional Euclidean space and $F(R^n)$ denotes the set of all fuzzy subsets on R^n. If $u \in F(R^n)$, $r \in (0,1]$, then we write $[u]^r = \{x \in R^n : u(x) \geq r\}$. Suppose that $u \in F(R^n)$, satisfies the following conditions:

(1) u is a normal fuzzy set, i.e., an $x_0 \in R^n$ exists such that $u(x_0) = 1$,
(2) u is a convex fuzzy set, i.e., $u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for any $x, y \in R^n$ and $\lambda \in [0,1]$,
(3) u is upper semicontinuous,
(4) $[u]^0 = \overline{\{x \in R^n : u(x) > 0\}} = \overline{\bigcup_{r\in(0,1]}[u]^r}$ is compact, where $\overline{A}$ denotes the closure of A.

Then, u is called a fuzzy number. We use E^n to denote the fuzzy number space [5,10,13,20].

It is clear that each $u \in R^n$ can be considered as a fuzzy number u defined by

$$u(x) = \begin{cases} 1, & x = u, \\ 0, & \text{otherwise}. \end{cases}$$

Theorem 2.1. *(See [9,13].) If $u \in E^n$, then*

(1) *$[u]^r$ is a nonempty compact convex subset of R^n for any $r \in (0,1]$,*
(2) *$[u]^{r_1} \subseteq [u]^{r_2}$, whenever $0 \leq r_2 \leq r_1 \leq 1$,*
(3) *if $r_n > 0$ and r_n converging to $r \in [0,1]$ is nondecreasing, then $\bigcap_{n=1}^{\infty}[u]^{r_n} = [u]^r$.*

Conversely, suppose that for any $r \in [0,1]$, an $A^r \subseteq R^n$ exists that satisfies (1)–(3) above, then a unique $u \in E^n$ exists such that $[u]^r = A^r$, $r \in (0,1]$, $[u]^0 = \overline{\bigcup_{r\in(0,1]}[u]^r} \subseteq A^0$.

For $u, v \in E^n$, $\alpha \in R$, the addition and scalar multiplication are defined by

$$(u+v)(x) = \sup_{s+t=x} \min\{u(s), v(t)\}, \tag{2.1}$$