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New error estimate in the iterative numerical method for nonlinear fuzzy Hammerstein–Fredholm integral equations

Shokrollah Ziari [a,*], Alexandru Mihai Bica [b]

[a] *Department of Mathematics, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran*
[b] *Department of Mathematics and Informatics, University of Oradea, Universitatii Street no. 1, 410087 Oradea, Romania*

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Abstract

In this paper, we obtain the error estimation of the iterative method based on quadrature formula to solve nonlinear fuzzy Fredholm integral equations of the second kind given in Fuzzy Sets & Syst. 245 (2014) 1–17, in terms of uniform and partial modulus of continuity. Moreover, we extend in the context of using the modulus of continuity, the notion of numerical stability of the solution with respect to the first iteration.

Keywords: Nonlinear fuzzy Hammerstein–Fredholm integral equations; Iterative numerical method; Uniform modulus of continuity; Partial modulus of continuity

1. Introduction

The concept of fuzzy set-valued integral was initiated by Dubois and Prade [12] and then investigated by Kaleva [21], Goetschel and Voxman [20], Nanda [26] and other authors. In [33], the Henstock-integral of fuzzy-valued functions is defined, while the fuzzy Riemann integral and its numerical integration were investigated by Wu in [34]. In [8], Bede and Gal introduced some quadrature rules for the integral of fuzzy-number-valued mappings. The interest on fuzzy Fredholm integral equations is based on its applications in fuzzy financial and economic systems (see [11]). Mordeson and Newman (see [25]) started the study of fuzzy integral equations using the concept of fuzzy linear transformation. The Banach fixed point principle and the Darbo's fixed point theorem are powerful tools to investigate the existence and the existence and uniqueness of the solution of fuzzy integral equations (see [6,7,17,30,31]).

There are various numerical methods developed for fuzzy integral equations. Up to present, these methods apply quadrature formulas (see [17,18]), Nyström techniques (see [1]), Adomian decomposition (see [2,5]), iterative techniques and successive approximations (see [8–10,14,15,17] and [29]). Other numerical methods developed for fuzzy integral equations use Bernstein polynomials (see [13] and [27]), Galerkin techniques (see [23]), Lagrange interpo-

* Corresponding author.
E-mail addresses: shok_ziari@yahoo.com (S. Ziari), abica@uoradea.ro (A.M. Bica).

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lation (see [16]), homotopy analysis (see [24]), homotopy perturbation (see [22]), divided and finite differences (see [28]), Legendre (see [32]) and Haar wavelets (see [35]).

Recently, in [10], Bica and Popescu developed an iterative numerical method to solve nonlinear fuzzy Hammerstein–Fredholm integral equations. In the present paper, we investigate the nonlinear fuzzy Hammerstein–Fredholm integral equation of the second kind:

$$F(t) = f(t) \oplus (FR) \int_a^b K(s,t) \odot H(s, F(s))ds, \quad a \leq s \leq t \leq b \tag{1}$$

where $K(s,t)$ is a positive crisp kernel function over the square $a \leq s, t \leq b$, and F, f, H are fuzzy-number-valued functions such that $f : [a,b] \to R_F$ and $H : [a,b] \times R_F \to R_F$ are supposed to be continuous. The convergence of the iterative numerical method proposed in [10] is based on the error estimation in the approximation of the solution of Eq. (1) that was obtained using supplementary Lipschitz conditions for f, K, and H. Moreover, in [10] it is introduced the notion of numerical stability with respect to the choice of the first iteration. As a main contribution, in this paper we obtain the error estimate in a more general case without supplementary Lipschitz conditions for f and K, and test for the first time the numerical stability on some numerical experiments. The error estimate obtained in this paper is expressed in terms of the modulus of continuity for f and K, that affect the statement concerning the numerical stability. These generate interesting considerations about expressing the numerical stability for iterative methods in various situations (that include the context of this paper and those from [10]) showing up its dependence by the involved quadrature formula and by the imposed conditions to f, K, and H. The paper is organized as follows: In Section 2, we review some elementary concepts of the fuzzy set theory and modulus of continuity used in the next sections. In Section 3, we point out some properties of the sequence of successive approximations and remember the iterative method to obtain the numerical solution of (1), firstly presented in [10]. The error estimation of the iterative method is obtained in Section 4 in terms of uniform and partial modulus of continuity, proving the convergence of the method. Since in [10] the error estimate is obtained using supplementary Lipschitz conditions, our result in this paper is obtained in a more general condition, using only the modulus of continuity. Section 5 includes the investigation of the numerical stability extended in terms of the modulus of continuity. In Section 6 we illustrate the iterative method of two numerical experiments testing the convergence and the numerical stability with respect to the choice of the first iteration. Some concluding remarks are given in the final section.

2. Preliminaries

Definition 1. (See [3].) A fuzzy number is a function $u : R \to [0, 1]$ having the properties:

1. u is normal, that is $\exists\, x_0 \in R$ such that $u(x_0) = 1$,
2. u is fuzzy convex set

$$(\text{i.e. } u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\} \quad \forall x, y \in R, \lambda \in [0,1]),$$

3. u is upper semi-continuous on R,
4. the set $\overline{\{x \in R : u(x) > 0\}}$ is compact.

The set of all fuzzy numbers is denoted by R_F.

In practice, more useful is the LU-representation of a fuzzy number given in [20].

Definition 2. (See [20].) An arbitrary fuzzy number is represented in parametric form (or in LU-representation) by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$, which satisfies the following requirements:

1. $\underline{u}(r)$ is a bounded left continuous non-decreasing function over [0, 1],
2. $\overline{u}(r)$ is a bounded left continuous non-increasing function over [0, 1],
3. $\underline{u}(r) \leq \overline{u}(r), \quad 0 \leq r \leq 1.$