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Migrative uninorms and nullnorms over t-norms and t-conorms

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Abstract

In this paper the notions of α-migrative uninorms and nullnorms over a fixed t-norm T and over a fixed t-conorm S are introduced and studied. All cases when the uninorm U lies in any one of the most usual classes of uninorms are analyzed, characterizing all solutions of the migrativity equation for all possible combinations of U and T and for all possible combinations of U and S. A similar study is done for nullnorms.

Keywords: Aggregation function; Uninorm; Nullnorm; Migrativity; t-Norm; t-Conorm

1. Introduction

The process of merging a given number of data into a representative value is usually carried out by the so-called aggregation functions. There are many fields where such a process is required at some step and this is the reason because aggregation functions have become an essential tool in many applications, from mathematics and computer science to economics and social sciences. Thus, the interest in aggregation functions has considerably grown in last decades and this interest is supported by the publications of some monographs entirely devoted to aggregation functions [4,8,21].

There are many different classes of aggregation functions and their election usually depends on the context where they are going to be applied. For instance, t-norms and t-conorms that generalize the logical connectives “AND” and “OR” of classical logic, are particularly important for their role in the theory of fuzzy sets and its applications [24]. In the same way, uninorms [20,38] that are a generalization of both, t-norms and t-conorms, have proved to be useful not only in this field, but also in many others like decision making, expert systems, fuzzy system modelling, aggregation and so on. Similarly, nullnorms [7,27] are also a generalization of t-norms and t-conorms and they are extremely related with uninorms.

One of the main topics in the study of aggregation functions from the theoretical point of view is directed towards the characterization of those that verify certain properties that may be useful in each context. The study of these

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properties for certain aggregation functions usually involves the resolution of functional equations [1]. One of these properties is *α-migrativity*, introduced in [14]. For any $\alpha \in [0,1]$ and a mapping $F:[0,1]^2 \to [0,1]$, this property is described as

$$F(\alpha x, y) = F(x, \alpha y) \quad \text{for all } x, y \in [0,1]. \tag{1}$$

The interest of this property comes from its applications, for example in decision making processes [6], when a repeated, partial information needs to be fusioned in a global result, or in image processing, since in this context migrativity expresses the invariance of a given property under a proportional rescaling of some part of the image [30]. Migrativity is also interesting from the theoretical point of view because of its relationship in the construction of new t-norms through convex combinations of two given ones [14,32,33].

The migrativity property (and successive generalizations) has been studied for t-norms in [17–19,31], for t-subnorms in [37], for semicopulas, quasi-copulas and copulas in [3,12,13,15,30] and for aggregation functions in general in [5,6,25,34]. Note that in Eq. (1) the product αx can be replaced by any t-norm T_0 obtaining the property for t-norms called (α, T_0)-migrativity, that can be written as

$$T\big(T_0(\alpha, x), y\big) = T\big(x, T_0(\alpha, y)\big) \quad \text{for all } x, y \in [0,1] \tag{2}$$

being T_0 a t-norm and $\alpha \in [0,1]$. This generalization of the migrativity for t-norms has been recently studied in [18,19]. By dualization, a similar definition can be given for t-conorms as it was pointed out in [28]. Moreover, this study has been extended to uninorms with the same neutral element in [29] (the particular case of representable uninorms was also solved in [5]). Let us also note that the migrativity equation (2), written for aggregation functions in general, is a particular case of the associativity general equation that was considered in [9] in the study of the consistency of n-ary recursive representations of aggregation functions from binary aggregation functions.

However, all the previous studies have a common point: they always deal with aggregation functions (t-norms, t-conorms or uninorms) having the same neutral element. This condition is not necessary to find out solutions of the migrativity property as we will see in the present work. As a first step in this direction, we will present in this paper a complete study of those uninorms with neutral element $e \in]0,1[$, and also nullnorms, that are α-migrative over t-norms and t-conorms.

The article is organized into different sections. After this introduction, we include a preliminary section to establish the necessary notation and we recall some basic definitions, specially on uninorms and nullnorms. In Section 3 we introduce the definition of (α, T)-migrative uninorm for a given t-norm T, analysing some of its initial properties. We continue with the characterization of those (α, T)-migrative uninorms, that lay in each one of the most usual classes of uninorms, i.e., uninorms in $\mathcal{U}_{\min}$ and $\mathcal{U}_{\max}$, idempotent uninorms, representable uninorms and uninorms continuous in the open square $]0,1[^2$. To end this section, an analogous study is done for uninorms that are (α, S)-migrative for a given t-conorm S, and we list the dual results for this case without proofs. In Section 4 we similarly deal with (α, T) and (α, S)-migrative nullnorms. We end the paper with a section of conclusions and future work.

2. Preliminaries

We will assume the basic theory of t-norms and t-conorms. The definitions, notations and results on them can be found in [2,24]. We will just give in this section some basic facts about uninorms and nullnorms. More details can be found in [7,10,20,23,27,36].

Definition 1. A binary function $U:[0,1]^2 \to [0,1]$ is called a *uninorm* if it is associative, commutative, non-decreasing in each variable and there is a neutral element $e \in [0,1]$ such that $U(e,x) = x$ for all $x \in [0,1]$.

Evidently, a uninorm with neutral element $e = 1$ is a t-norm and a uninorm with neutral element $e = 0$ is a t-conorm. For any other value $e \in]0,1[$ the operation works as a t-norm in $[0,e]^2$, as a t-conorm in $[e,1]^2$ and its values are between minimum and maximum in the set of points $A(e)$ given by

$$A(e) = [0,e[\times]e,1] \cup]e,1] \times [0,e[.$$

We will usually denote a uninorm with neutral element e and underlying t-norm and t-conorm, T and S, by $U \equiv \langle T, e, S\rangle$. For any uninorm it is satisfied that $U(0,1) \in \{0,1\}$ and a uninorm U is called *conjunctive* if $U(1,0) = 0$ and *disjunctive* when $U(1,0) = 1$. On the other hand, the most studied classes of uninorms are: