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From fuzzy data analysis and fuzzy regression to granular fuzzy data analysis

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Abstract

This note offers some personal views on the two pioneers of fuzzy sets, late Professors Hideo Tanaka and Kiyoji Asai. The intent is to share some personal memories about these remarkable researchers and humans, highlight their long-lasting research accomplishments and stress a visible impact on the fuzzy set community.

The note elaborates on new and promising research avenues initiated by fuzzy regression and identifies future developments of these models emerging within the realm of Granular Computing and giving rise to a plethora of granular fuzzy models and higher-order and higher-type granular constructs.

Keywords: Fuzzy set pioneers; Fuzzy data analysis; Fuzzy regression; Granular Computing

Professors Hideo Tanaka and Kiyoji Asai are the pioneers, innovators, academicians, who have made so many outstanding contributions to the fundamentals, practice, and technology of fuzzy sets.

Their contributions to the area of fuzzy sets are truly remarkable and multifaceted. They were at the forefront of fuzzy sets worldwide when this new research was at its inception stage. In Japan, they shaped the landscape of the fuzzy set technology, which in the eighties emerged as one of household names of intelligent systems and gave rise to a wave of Machine Intelligence Quotient product or in brief MIQ-products (both home appliances and industrial setups), which started to exhibit a certain level of intelligence (expressed in terms of MIQ, to be more specific). They provided mentorship to numerous brilliant Ph.D. students who occupy now prominent academic positions across Japan and worldwide; just to mention Professors Junzo Watada, Koji Izumi, Masahiro Inuiguchi, Hisao Ishibuchi, Peijun Guo, and Tomoe Entani, among others.

From my personal contacts with them, I can vividly recall interesting enlightening meetings, witty comments, and interesting perspectives they always brought into discussion no matter if it was of technical matter or dealt with some

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general issues. They have a rare talent to listen, motivate, offer an innovative perspective and deliver friendly guidance and advices.

Their research legacy is highly important, influential but also thought provoking by opening new research horizons. In this brief note, I would like to make a point that one of their ideas, fuzzy regression, can be seen as a genuine stimulus to the development of the recent area known as Granular Computing [1,2,5].

In their seminal paper [3], Professors Tanaka and Asai brought forward an innovative and far reaching concept of fuzzy regression. The underlying idea, as we see it today, is lucid, convincing and practically relevant: a traditional regression problem is innovatively reformulated in the language of fuzzy sets. The parameters of the regression models are now fuzzy numbers. Numeric input–output data are modeled by a fuzzy function $Y = f(\boldsymbol{x}, \boldsymbol{A})$ where $\boldsymbol{A}$ is a vector of parameters of the fuzzy regression modeled by triangular or trapezoidal fuzzy numbers. For a linear type of the fuzzy function, fuzzy regression is built by solving a linear programming (LP) program. But the concept extends far beyond the notion of regression models. By looking at the essence of the problem more thoroughly, one has to acknowledge that modeling *numeric* data through *fuzzy* models opens up a new avenue of innovative thinking and is very much in line with the incompatibility principle coined by Zadeh in [4].

A direct follow-up of this observation, is modeling of numeric data in the setting of information granules subsequently giving rise to a new direction of *granular* modeling and granular fuzzy modeling, in particular.

Information granularity and information granules play an important role in system modeling. There are a number of interesting and practically legitimate design and application scenarios where the inherent granularity of the models becomes visible and plays an important role. We briefly highlight the main features of these modeling environments.

Granular characterization of models It is needless to say that there are no ideal models which can capture the data without any modeling error meaning that the output of the model is equal to the output data for all inputs forming the training data. To quantify this lack of accuracy, we give up on the precise numeric model (no matter what particular format it could assume) and make the model granular by admitting granular parameters and allocating a predetermined level of granularity to the respective parameters so that the granular model obtained in this way "cover" as many training data as possible. The granular characterization applies not only to fuzzy models but embraces a number of interesting pursuits as reported in the literature, say Z-numbers [6].

Emergence of granular models as a manifestation of transfer knowledge Let us consider that for a current problem at hand we are provided with a very limited data set – some experimental evidence (data) $\boldsymbol{D}$ expressed in terms of input–output pairs. Given this small data, two possible scenarios could be envisioned:

(a) we can attempt to construct a model based on the data. As the current data set is very limited, designing a new model does not look quite feasible: it is very likely that the model cannot be constructed at all, or even if formed, the resulting construct could be of low quality.

(b) we would like to rely on the existing model (which although deals with not the same situation but has been formed on a large and quite representative body of experimental evidence). We may take advantage of the experience accumulated so far and augment it in a certain sense so that it becomes adjusted to the current quite limited albeit current data. In doing this, we fully acknowledge that the existing source of knowledge has to be taken with a big grain of salt and the outcomes of the model have to be reflective of partial relevance of the model in the current situation. We quantify this effect by making the parameters of the model granular (viz. more abstract and general) so that one can build the model around the conceptual skeleton provided so far. In this case, viewing the model obtained so far as a sound source of knowledge, we are concerned with a concept of an effective knowledge transfer, see Fig. 1. The knowledge transfer (which, in essence, is represented by some model) manifests in the formation of a more abstract version of the original model.

Granular models as a result of model reduction Models, especially those with a modular structure can give rise to their granular counterparts. This could be a result of model reduction: the reduced structure of the model is made granular using which we compensate for this reduction. This effect is visible quite vividly in case of rule-based models or fuzzy rule-based models. Let us recall that those are the models composed by "P" rules of the form

$$\text{– if condition is } A_i \text{ then conclusion is } B_i \tag{1}$$