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Foundational contributions of K. Asai and H. Tanaka to fuzzy optimization

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Abstract

The first researchers to publish an operationalization of the Bellman and Zadeh's original approach to "Decision making in a fuzzy environment" [1] were H. Tanaka and K. Asai in their 1973 article [2] which was published in English in 1974 [3]. These two researchers developed many fundamental theories and methods including novel concepts of fuzzy operations research [4] with their collaborators. They went beyond the original proposal for fuzzy optimization set forth by Bellman/Zadeh. They are important founders of fuzzy optimization. This paper gives an overview of their great contributions to fuzzy optimization.

Keywords: Fuzzy linear programming; Decision making; Possibility theory; Fuzzy number

1. Introduction

In 1984, Professors K, Asai and H. Tanaka first published in the newly created journal *Fuzzy Sets and Systems*, founded in 1978. This 1984 article [5] was a synthesis and extension of their research into fuzzy set theory and fuzzy optimization that began in 1967. It is thus natural to devote this article in this special issue of Fuzzy Sets and Systems to the work of these late pioneers.

The earliest days of the fuzzy sets and systems, as many founders of fuzzy set theory have expressed, including Professors K. Asai and H. Tanaka, researchers in the related fields were negative and even hostile toward fuzzy set theory. These pioneers fought hard against the prejudices directed toward fuzzy sets by faithfully and positively developing the mathematical tools and applications, and by convincing and sometimes heated discussions at congresses, in scientific papers, as well as in a formal debate [6] in Bordeaux, France, 1972.

Professor Kiyoji Asai is considered to be the first fuzzy set theory researcher in Japan and so the father of fuzzy set theory in Japan. He worked with Professor L.A. Zadeh at the University of California, Berkeley for a year in 1967. Professor Asai published a paper on fuzzy automata with Professor Kitajima in 1970 [7], followed by other

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ones in 1971 [8,9], 1972 [10] and 1974 [11] and for this reason he is known as the first Japanese researcher to have written a journal article on fuzzy sets and systems. He founded a group to study fuzzy sets and systems in Japan by establishing a "Research Group of Fuzzy Sciences" in 1980 in the western part of Japan. The research group is one of the predecessors of SOFT (Japan Society for Fuzzy Theory and Intelligent Informatics).

In 1967, Professor Hideo Tanaka, who was a Ph.D. student of Professor Asai, and two fellow students, Professor Tsutsui and Professor Watada found and studied Professor Zadeh's first paper on fuzzy sets published in 1965 [12] and discussed it with Professor Asai. So Professor Tanaka can be thought of as the person who interested Professor Asai in fuzzy set theory. From this time until Professor Asai's retirement in 1986, they (Tanaka–Asai) worked together. Professor Tanaka also visited Professor Zadeh at the University of California, Berkeley from 1972 to 1973. In 1973 [2] and 1974 [3], together with Professor Asai and Professor Okuda, who was then a Ph.D. student, he published a paper on fuzzy mathematical programming, which is a seminal contribution to fuzzy optimization. After this paper, Professor Asai and Professor Tanaka developed approaches to management sciences we now know as "Fuzzy Operations Research" [4].

This article presents an overview of Professor Asai and Tanaka's contributions restricting itself to their papers on fuzzy optimization. They published many more papers on various other topics in fuzzy operations research including fuzzy decision analysis, fuzzy regression, fuzzy data analysis, fuzzy Analytic Hierarchical Processes (AHP), fuzzy Data Envelopment Analysis (DEA). The latter is a fuzzy set theory based approach which uses fuzzy mathematical programming (linear) to determine a relative performance of organizational units where the presence of multiple inputs and outputs makes comparisons difficult (see, for example, http://www.deafrontier.net/deaintro.html). In what follows, we write Professor Asai's and Tanaka's names without their titles. In Section 2, we describe their pioneering work on flexible programming. In Section 3, we review their contributions to linear programming with fuzzy numbers. Two models are explained: one is based on the nonnegativity of fuzzy numbers and the other is based on the 'fuzzy max' operation. In Section 4, we describe the first proposal of fuzzy solutions as well as their later developments. We note that fuzzy solutions and their formulation of linear programming problems with fuzzy numbers are equivalent in that one can be obtained from the other by the exchange of fuzzy coefficients with fuzzy decision variables. The value of information of a fuzzy linear programming problem is explained in Section 5. In Section 6, we describe Tanaka's approaches to fuzzy portfolio selection problems using exponential possibility distributions since not only is the process by which one obtains an exponential possibility distribution a mathematical programming problem, portfolio analysis are non-linear programs themselves. Moreover, the exponential possibility distributions are also applied to fuzzy solutions in linear programming problems with fuzzy resource constraints. In Section 7, we conclude this paper with a brief comment on their other contributions.

Before we begin describing some details of the contributions of Asai/Tanaka to fuzzy optimization, we wish to briefly indicate the significance of this contribution to fuzzy and possibilistic optimization. In the period 1967–1973, it was difficult to find a way to translate the original idea developed by Bellman/Zadeh of optimization in a fuzzy environment into a mathematical framework that had concrete meaning in that it fits into a theory in which its operationalization led to solutions that made sense within the context of the framework itself. What does this mean? When one looks what Bellman/Zadeh proposed as a criterion and one looks at a general optimization problem set in a fuzzy environment, how does one actually: (1) Write down a fuzzy objective function, (2) Write down a fuzzy constraint, (3) Determine mathematically what equality means in the presence of fuzzy entities, (4) Put 1–3 into meaningful equations, (5) Solve the problem (obtain an algorithm)? Asai/Tanaka were the first to find a way. To be sure, their approach [2,3,13] was more complicated and computationally slower than other approaches that arose soon after, namely [14], the contents of which were in a conference proceeding two year before. Nevertheless, Asai/Tanaka opened the field. Secondly, their continued research into various mathematical areas involving fuzzy optimization (AHP, DEA, Linear Regression, Portfolio Section, Operations Research) showed the way for a new generation of researchers. Lastly, the mark of Professor Asai and Professor Tanaka on the professional and personal lives of this succeeding generation of researchers can be seen in their amazing accomplishments that have gone beyond their professors' expectations and dreams. Professor Asai and Tanaka have been amazingly generous and prolific in their joint research. *None* of their research papers listed in the bibliography is single authored. All are with their students and junior staff.