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ScienceDirect

Fuzzy Sets and Systems 274 (2015) 62–78

FUZZY sets and systems

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A note on rationality conditions of fuzzy choice functions ☆

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Received 18 September 2013; received in revised form 25 July 2014; accepted 29 July 2014

Available online 14 August 2014

Abstract

This note investigates the rationality conditions of fuzzy choice functions. For studying the rationality of fuzzy choice functions, Banerjee introduced many regularity conditions that are defined for revealed preference relations. However, in the standard framework of economics and decision-making theory researchers usually assume people's choice behavior is observable, while the preference (behind the choice behavior) is unobservable. It is worthwhile to study rationality conditions that are defined for fuzzy choice functions instead that for revealed preference relations. For this purpose, we propose a set of new rationality conditions depending only on the fuzzy choice function. We study the connection among these rationality conditions and give a relatively complete description for them. We also find that Banerjee-rationality is too weak to capture some kind of consistency of the fuzzy choice function. This motivates us to introduce two notions on strong rationality. The relationships between our new consistency conditions and the rationality (Banerjee-rationality and strong rationality) of the fuzzy choice function are discussed.

Keywords: Rationality; Fuzzy revealed preference relation; Fuzzy choice function

1. Introduction

Preference relations, choice functions and the related rationality conditions have been studied by economists and decision-theorists for a long time. In the crisp setting, a choice function is just a rule by which one or more elements are selected from a set of available alternatives. In this field, the rationality of a choice function attracted extensive attention. For studying the rational behavior of a consumer, Samuelson [14] introduced the concept of revealed preference and proposed the Weak Axiom of Revealed Preference (WARP). Houthakker [9] introduced the Strong Axiom of Revealed Preference (SARP) as a stronger property. Afterwards, more conditions were proposed to characterize the rationality of choice functions, among which, the most popular are the Strong Congruence Axiom (SCA) by Richter [13], the Weak Congruence Axiom (WCA) by Sen [15], the Conditions α, α_2, β, γ, γ_2 by Sen [15,16], etc.

☆ Research was supported in part by the National Natural Science Foundation of China (Nos. 11171084, 51176044), the Natural Science Foundation of Zhejiang Province (Nos. LY13A010018, LQ12F03003).
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http://dx.doi.org/10.1016/j.fss.2014.07.023
0165-0114/

Meanwhile, the relations among these rationality conditions were studied and a lot of fundamental results were obtained. For example, Arrow [1] studied the relationships between WARP, SARP, α, etc. Sen [15,16] investigated the relationships among these conditions more systematically and proved the so-called Arrow–Sen theorem. For more investigations of the connection among rationality conditions of choice functions, we refer the readers to [4,18,19].

The classical choice functions and revealed preference theory characterize deterministic phenomena. However, in the real world there are economic and social phenomena which do not always comply with precise rules. Fuzzy set theory provides an appropriate tool to model vague preferences and vague choices. Particularly, the vague preferences can be represented by fuzzy preference relations and vague choices by fuzzy choice functions.

In fuzzy environment, Orlovsky [12] initially studied the crisp choice functions with fuzzy preference relations. From then on, many researchers discussed the problems of generating crisp choice functions from fuzzy preference relations and rationalizing crisp choice functions with fuzzy preference relations (see, for instance, [2,5,11,17]). However, as Banerjee [3] pointed out: "if the preferences are permitted to be fuzzy, it seems natural to permit the choice function to be fuzzy as well. This also tallies with experience". Based on this argument, Banerjee [3] introduced the concept of fuzzy choice function and studied the rationality conditions of fuzzy choice functions and fuzzy preference relations. Banerjee [3] established two Weak Axioms of Fuzzy Revealed Preference, WAFRP1 and WAFRP2, and fuzzy congruence conditions, FC1, FC2 and FC3. Wang [20] showed that FC1 implies FC2 and FC3 implies FC2, while FC1 and FC3 are independent conditions. Lately, Georgescu [6–8], Martinetti et al. [10] and Wu et al. [21] studied the fuzzy choice function and related rationality conditions more deeply. Georgescu generalized the definition of Banerjee's fuzzy choice function by extending the available domain to fuzzy sets and fuzzified some crisp rationality conditions such as WCA, SCA, WARP, SARP, α, β etc. She also investigated the relations among these conditions and extended them from crisp setting to fuzzy setting. More recently, Martinetti et al. [10] dealt with fuzzy rationality conditions and studied the connection between them extensively. They generalized many of Georgescu's results and investigated the connection between fuzzy rationality conditions under weaker conditions. In the framework of Banerjee's choice function, Wu et al. [21] gave a further study on rationality conditions under the assumption of normal choice set.

In this note, we study rationality conditions in the framework of Banerjee's fuzzy choice function. Analyzing the consistency conditions (e.g. FC1, FC2, FC3, Condition A, Condition B, SAFRP1, SAFRP2, WAFRP1, WAFRP2) proposed by Banerjee [3], we can find that they are defined for fuzzy revealed preference relations. As we know, preference and choice are different mathematical objects, though the two concepts are closely related. In the standard framework of economics and decision-making theory researchers usually assume people's choice behavior is observable but the preference is unobservable. And accordingly, as a standard approach, researchers usually investigate people's preference by means of learning their choice behavior. It is reasonable and realistic to investigate the rationality through the lens of the observable choice behavior. So it is worthwhile to present rationality conditions that are defined for choice functions rather than for revealed preference relations. In the crisp setting, many rationality conditions defined only on choice functions were established, such as the so-called expansion–contraction conditions $\alpha, \beta, \gamma, \delta$ (see, for instance, [16]). In the fuzzy setting, Georgescu [8] and Wu et al. [21] proposed the contraction–expansion rationality conditions $F\alpha$, $F\beta$, $F\delta$ and $F\gamma$, respectively. These conditions are defined for fuzzy choice functions involving the continuous t-norms.

In this paper we investigate some new rationality conditions defined for fuzzy choice functions. We study the connection between the existing and our new rationality conditions. We also find that the Banerjee-rationality is too weak to capture some kind of consistency of the fuzzy choice function. This motivates us to introduce two notions on strong rationality. The relationships between our new consistency conditions and the rationality (Banerjee-rationality, strong rationality) of the fuzzy choice function are discussed. Furthermore, we improve some results obtained by Wang in [20].

The rest of this paper is organized as follows. In Section 2, we recall some preliminaries on the choice functions and preference relations. In Section 3, we propose some new rationality conditions defined for fuzzy choice functions. We study the relationships among the rationality conditions in Section 4. The relationships between the rationality conditions and the rationality of the fuzzy choice function are discussed in Section 5. Some counterexamples are constructed in Section 6. The last section provides the concluding remarks.