Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 274 (2015) 79–96

FUZZY sets and systems

www.elsevier.com/locate/fss

Pairwise comparison based interval analysis for group decision aiding with multiple criteria

Tomoe Entani [a], Masahiro Inuiguchi [b,*]

[a] *Graduate School of Applied Informatics, University of Hyogo, Kobe, Hyogo 650-0047, Japan*
[b] *Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan*

Received 30 September 2013; received in revised form 28 December 2014; accepted 2 March 2015

Available online 10 March 2015

Abstract

Interval AHP (Analytic Hierarchy Process) was proposed to obtain interval weights from a given pairwise comparison matrix showing relative importance between criteria. In this paper, Interval AHP is applied to group decision problems. Interval AHP is first revised suitably for comparing alternatives from the viewpoint that the interval weight vector shows the set of agreeable weight vectors for the decision maker. Under individual interval weight vectors obtained from individual pairwise comparison matrices, three approaches to obtaining a consensus interval weight vector are proposed. One is the perfect incorporation approach that obtains consensus interval weight vectors including all individual interval weight vectors. By this approach, we can count out indubitably inferior alternatives. The second is the common ground approach that obtains consensus interval weight vectors included in all individual interval weight vectors. By this approach we can find agreeable group preference between alternatives when all individual opinions are similar. The third is the partial incorporation approach that obtains consensus interval weight vectors intersecting all individual interval weight vectors. By this approach we can find agreeable group preference between alternatives when individual opinions are not similar. The usefulness of the proposed three approaches is demonstrated by simple numerical examples.

Keywords: Analytic hierarchy process; Group decision-making; Interval analysis; Linear programming; Dominance relation

1. Introduction

AHP (Analytic Hierarchy Process) is an approach to multi-criteria decision making problems and induces the preference of a decision maker from his/her intuitive judgements [1]. In AHP, the decision problem is structured hierarchically as criteria and alternatives. At each node except leaf nodes of the hierarchical tree, a weight vector for criteria or for alternatives is obtained from a pairwise comparison matrix given by a decision maker. The overall priority of an alternative is obtained as the weighted sum of the local weights. As all priorities are estimated by real

* Corresponding author.
E-mail addresses: entani@ai.u-hyogo.ac.jp (T. Entani), inuiguti@sys.es.osaka-u.ac.jp (M. Inuiguchi).

http://dx.doi.org/10.1016/j.fss.2015.03.001
0165-0114/

numbers, a weak order of alternatives is obtained. However, the pairwise comparison matrix given by a decision maker is not very accurate and includes inconsistency. In AHP, the consistency index is defined and calculated for each pairwise comparison matrix. The resulting weight vector is accepted when the value of the consistency index is in the acceptable range.

To treat the imprecision or vagueness of human judgement, intervals and fuzzy numbers were applied to the expression of the pairwise comparison matrix. van Laarhoven and Pedrycz [2] treated a pairwise comparison matrix with fuzzy components. They applied the logarithmic least squares method to obtain a fuzzy weight vector from the fuzzy pairwise comparison matrix. This original method has some problems in the normalization and some modifications were proposed [3,4]. Buckley [5] also proposed fuzzy hierarchical analysis using a pairwise comparison matrix with fuzzy components. He extended the geometric mean method to fuzzy case to obtain a fuzzy weight vector. Later, Buckley et al. [6] and Csutora and Buckley [7] proposed the fuzzification of the maximum eigenvalue method for obtaining a fuzzy weight vector. Through those approaches, alternatives are evaluated by fuzzy priorities. The ranking of alternatives is done by a method for ranking fuzzy numbers.

On the other hand, Saaty and Vargas [8] considered the interval judgement in a pairwise comparison matrix and investigated its effect on the stability of the rank order of alternatives. Arbel [9] treated the interval judgements in pairwise comparison as a range that true relative importance exists. He translated the interval pairwise comparison matrix to linear constraints on weight vectors. From the region expressed by the linear constraints, a consistent normalized weight vector is selected. Weight vector selection methods from the region have been proposed in [10,11]. However, this approach may have a problem when there is no consistent normalized weight vector in the region obtained from the interval pairwise comparison matrix (see [12]). Islam et al. [13] proposed a goal programming approach to obtaining a normalized weight vector from an inconsistent pairwise comparison matrix with interval components. Yu [14] modified this approach to obtain a normalized weight vector by balanced evaluation of deviations from the given interval data using logarithmic transformation of weights. Mikhailov [15,17] and Mikhailov and Tsvetinov [16] introduced tolerances to the linear constraints induced from a pairwise comparison matrix with interval components and proposed a fuzzy programming approach to obtaining a normalized weight vector. Dopazo et al. [18] extended this type of approach to the case of pairwise comparison matrices with fuzzy components. A fuzzy component is transformed to a nested intervals by a finite collection of α-cuts. Then linear constraints for all α-cuts are considered in the logarithmic transformed weight vector space. A goal programming approach is applied to obtaining a normalized weight vector. Those approaches express the uncertain judgement in pairwise comparison explicitly by intervals or by fuzzy numbers but a crisp normalized weight vector suitable for the given uncertain pairwise comparison is obtained.

Sugihara and Tanaka [19] proposed an approach to obtaining an interval weight vector from crisp pairwise comparison matrix. This approach is called Interval AHP and based on the idea that the decision maker does not perceive a precise weight vector but a range of weight vectors vaguely in his/her mind. Any weight vector in the range is considered acceptable for the decision maker. Each pairwise comparison by the decision maker is assumed to be done by arbitrarily selected weights from the ranges. This arbitrary selection for each comparison is considered the cause of the inconsistency of the pairwise comparison matrix. An interval weight vector is estimated to include the given pairwise comparison matrix as a possible realization. Later Sugihara et al. [20] extended Interval AHP to treat the pairwise comparison matrix with interval components. Unlike the aforementioned approaches to interval pairwise comparison matrix, interval weight vectors are estimated. In the sense that the interval weight vector is estimated from an interval pairwise comparison matrix, this approach is similar to the fuzzy AHP estimating a fuzzy weight vector from a fuzzy pairwise comparison matrix, but estimation ideas are different. Two interval weight vectors are obtained as the outer and inner estimations in Interval AHP while a fuzzy weight vector is obtained as an approximate estimation in fuzzy AHP. In Interval AHP, the alternatives are ranked by the obtained interval weight vector and the dominance relation between alternatives is often a preorder. Interval AHP approaches can be seen as an AHP counterpart of Tanaka's fuzzy/interval linear regression analysis [21–23].

This paper extends Interval AHP to group decision making. Namely, we investigate methods for inducing an interval weight vector from m individual crisp pairwise comparison matrices and for finding the consensus interval weight vector.

AHP approaches were used to support the interpersonal information exchange [24,25] in group decision making problems. There are two basic approaches for aggregating individual opinions into a group opinion [26]. One approach first aggregates the individual judgements of pairwise comparisons and then provides a consensus weight vector. This approach is based on the assumption that the group is a synergistic unit and called the comparison matrix aggregation