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Extended procedure for computing the values of the membership function of a fuzzy solution to a class of fuzzy linear optimization problems

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Abstract

In 1994, Chanas and Kuchta suggested one approach to determine the membership function of a fuzzy solution to the fuzzy linear optimization problem with fuzzy coefficients in the objective function, based on computing the sum of lengths of certain intervals. In 2012, Dempe and Ruziyeva introduced a methodology for realizing Chanas and Kuchta's idea, and derived explicit formulas for computing the endpoints of the suggested intervals in the particular case of triangular fuzzy numbers. The purpose of this paper is to extend Dempe and Ruziyeva's approach by handling the possible degeneracy of basic feasible solutions, and derive new formulas for computing the values of the membership function. The special example considered by Dempe and Ruziyeva is again used to illustrate the relevance of the extended solving procedure and new formulas.

Keywords: Fuzzy mathematical programming; Membership function; Bi-objective optimization; Pareto optimal solution; Linear optimization

1. Introduction

The class of fuzzy linear programming problems with fuzzy coefficients in the objective function is addressed in this paper. Cadenas and Verdegay [3] presented the most important models and methods for fuzzy linear programming. They analyzed the linear programming problems with fuzzy costs, fuzzy constraints and fuzzy coefficients in the constraint matrix as a unit. Many particular linear programming problems that involve fuzzy entities and their solving approaches can be found in the recent literature (see for instance [7,8,10], and [11]).

Dempe and Ruziyeva [7], applied the concept of fuzzy solution to the fuzzy linear optimization problem with fuzzy coefficients in the objective function and crisp constraints. Their methodology is the realization of the idea suggested by Chanas and Kuchta [4], and it is based on α-cuts and on the set of Pareto optimal solutions to a certain bi-objective optimization problem.

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The membership function approach introduced in [7] was used in [2] to develop an algorithm for the computation of optimal tolls on a traffic network, and in [13] to solve the fuzzy linear fractional optimization problem with fuzzy coefficients in the objective function.

In this paper we extend Dempe and Ruziyeva's procedure for computing the values of the membership function of the fuzzy solution, and rectify their formulas in the special case of triangular fuzzy numbers. The extension we propose takes into consideration the possible degeneracy of the basic solutions to a crisp optimization problem. Ehrgott [9] referred to the concept of degeneracy of a basis and showed why degeneracy is 'problematic' in the context of multi-criteria optimization problems.

When the special case of triangular fuzzy numbers was considered in [7], the deficient discussion of the optimality conditions generated an incorrect step in the direct computation of the values of the membership function of the fuzzy solution. In our approach we transform equivalently the optimality conditions, and rewrite completely the procedure for computing the values of the membership function. Comparing to Dempe and Ruziyeva's procedure [7], our computation algorithm is simpler, since it does not involve solving any additional optimization problem.

The rest of the paper is organized as follows: Section 2 contains a brief description of the fuzzy linear optimization problem with fuzzy coefficients in the objective function, and a solution concept. Our extended solving procedure for such problem is described in Section 3. Section 4 presents our new algorithm that explicitly computes the values of the membership function of the fuzzy solution, in the particular case of triangular fuzzy numbers. In Section 5, the solution to one instance of a traffic problem with fuzzy cost coefficients is given in extenso. Finally, Section 6 is devoted to conclusions and future works.

2. Preliminaries

For the convenience of the reader we repeat the relevant information from [7], thus making our exposition self-contained. Dempe and Ruziyeva investigated the fuzzy linear optimization problem

$$\begin{aligned} \min \quad & \widetilde{c}^T x \\ \text{s.t.} \quad & Ax = b, \\ & x \geq 0, \end{aligned} \tag{2.1}$$

where $\widetilde{c}$ is a vector of fuzzy numbers representing the coefficients of the objective function, A is the $m \times n$ matrix of the constraints, and $b \in R^m$ is the right-hand side vector of the constraints.

First, they replaced Problem (2.1) by the interval optimization problem

$$\begin{aligned} \min \quad & \left[c_L^T(\alpha)x, c_R^T(\alpha)x\right] \\ \text{s.t.} \quad & Ax = b, \\ & x \geq 0, \end{aligned} \tag{2.2}$$

where $[c_L^T(\alpha)x, c_R^T(\alpha)x]$ is the α-cut interval of the original fuzzy objective function.

Using the ordering of intervals proposed by Chanas and Kuchta in [5], Dempe and Ruziyeva next replaced Problem (2.2) by the bi-objective optimization problem

$$\begin{aligned} \min \quad & c_L^T(\alpha)x, \\ \min \quad & c_R^T(\alpha)x, \\ \text{s.t.} \quad & Ax = b, \\ & x \geq 0. \end{aligned} \tag{2.3}$$

Let $\psi(\alpha)$ denote the set of Pareto optimal solutions to Problem (2.3) for a certain α-cut. Using $\psi(\alpha)$, $\alpha \in [0, 1]$, they described the optimality of feasible solutions to the original problem (2.1). They also calculated the value of the membership function of the fuzzy solution at a given feasible solution $\bar{x}$, as the length of the interval of those values of α for which $\bar{x}$ is an efficient solution to (2.3), i.e.

$$\mu(\bar{x}) = \text{length}\left\{\alpha \in [0, 1] \,\middle|\, \bar{x} \in \psi(\alpha)\right\}. \tag{2.4}$$

Unfortunately, they wrote 'card' (and mentioned 'cardinality') instead of 'length' in (2.4), but the correctness of the computation was not affected.