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Non-additive interval-valued F-transform

O. Strauss

LIRMM Université Montpellier II, 61 rue Ada, 34392 Montpellier cedex 5, France

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Abstract

This article proposes a new interval-valued fuzzy transform. Its construction is based on a possibilistic interpretation of the partition on which the fuzzy transform is built. The main advantage of this approach is that it provides specific interval valued functions whose interpretation is straightforward. This interpretation relates to a traditional sampling/reconstruction framework where little is known about the sampling and/or reconstructing kernels. Numerous properties of the proposed approach are proved that could be useful for function analysis and comparison. In the experimental section, we illustrate some properties of the proposed transform while highlighting interesting features of the obtained framework.

Keywords: Choquet integral; Fuzzy transform; Sampling; Summative and maxitive kernels; Possibility theory; Confidence measures

1. Introduction

Nowadays, signal processing is mainly achieved through algorithms on discrete and quantified representations of real signals. In this domain, transformations are essential to transpose a signal to another space in order to obtain a more compact and meaningful representation of the signal. Transformations are extensively used for analysis, compression, encryption, filtering, inversion, information retrieval, etc. Many transformations have been proposed in the relevant literature. Fourier and Laplace transforms are the most widely used, which associate a complex decomposition with any real signal in the frequency domain. The advent of the fast Fourier transform (FFT) enabled real-time computation of the Fourier transform and thus its wide use in many applications. The discrete cosine transform can be seen as a simplification of FFT that only keeps the real part of the Fourier transform, thus associating a real decomposition with a real signal. It became popular through its use in the jpeg compression method. The wavelet transform was more recently proposed as a better solution for analyzing signals having compact support. In image processing, more dedicated transformations have also been proposed. For example, the Hough transform places the image in a parametric space, thus facilitating the retrieval of specific parametrized features (e.g. lines, curves, etc.). Its close relative, the Radon transform, has been proposed to solve the problem of reconstructing an image from its projections. Sampling and subsampling can also be seen as transformations. Sampling consists of associating a bounded set of real-valued

E-mail address: strauss@lirmm.fr.

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samples with a continuous signal, while subsampling associates a reduced number of representative samples with a real high resolution discrete signal. The idea underlying working with a sampled (or subsampled) signal is to use an inherent redundancy of the signal, so as to make it possible to work with the original signal by only manipulating a reduced set of samples. It is used to solve continuous problems by finite computation or to reduce the computation complexity of a signal processing algorithm. Other transforms exist like those of Hilbert, Gabor, Zack, etc. Most of those transformations consist of convolving the real signal with an appropriate set of kernels, which is the basis of the transform.

The fuzzy transform (or F-transform) recently proposed by Irina Perfilieva [23] belongs to this transform family. It consists of associating, with an original continuous or discrete real signal, a reduced set of real samples by projecting this signal on a fuzzy partition *à la* Ruspini [29]. This has drawn a great deal of attention from the scientific community since it is one of the rare uses of the fuzzy framework to directly handle real functions without any linguistic interpretation. It has been used for data analysis [26], compression (see references in [23]), segmentation [17], coding [16], solving differential equations, forecasting [18], scheduling [12], trading [33], etc. A special issue of this journal was recently dedicated to advances in fuzzy transform theory and applications [27].

The fuzzy transform involves two operations: a direct fuzzy transform (F-transform) which is the decomposition itself and an inverse fuzzy transform (IF-transform) that goes from the sampled space to the original space. The word "inverse" may seem somewhat inappropriate since applying the IF-transform to the F-transform of a signal leads to an estimate that is not equal to the original signal. However, as shown in [23], an appropriate choice of fuzzy partition can make the reconstructed signal an approximation of the original signal with any arbitrary precision.

With most transforms, the question arises as to the existence of an inverse transform, i.e. *is it possible to reconstruct the original signal from its transformation*? Except for the wavelet transform, the answers to this question given by the authors of most of the aforementioned transforms are highly debatable when applied to numerical signal processing. More precisely, if a signal is represented by a reduced number of values, the reconstruction of the original signal is risky. For example, the discrete Fourier transform has an inverse, i.e. it is possible to exactly reconstruct the original sampled signal from its transformed values. However, manipulations in the transformed space are often meaningless in the original space since the Fourier transform of a discrete signal is continuous while the FFT associates a discrete representation with a discrete signal. The Radon transform also has an inverse form in the continuous domain. However, this inverse form does not exist in the discrete space and a certain number of dedicated tools, including regularizations and optimizations, are required to invert a discrete Randon transform.

The inverse transform issue also naturally exists within the F-transform framework. The first proposition of Perfileva was to use the same shape function as that used to generate the partition to achieve both F- and IF-transforms. She proved that the obtained reconstruction locally minimizes a L_2 distance between the original and the reconstructed signal. However, as shown first by Crouzet [2] and then by Patané [21], this local criterion is not very relevant from a signal processing standpoint. Thus, in this setting, another basis should be preferred that leads to minimizing a global L_2 distance. As shown in [3], this leads to the least square interpolation procedure conventionally used in signal processing. However, this approach only applies to discrete functions.

Other techniques have been proposed to enhance the ability of the fuzzy transform framework to work with a simple representation of a signal. For example, Bede and Rudas [1] question the shape function of the weighting fuzzy numbers used to form fuzzy partitions. It appears, from a qualitative comparison, that the optimality of a particular shape function highly depends on the function to be represented. The partition can thus be adapted to the signal, as shown by Sefanini in [31]. The position of the partition nodes can also be adapted to have a higher concentration of atoms where the signal has more variations. However, adapting the partition to signals can require the use of two different partitions for two different signals, thus limiting the ability of using fuzzy transforms for combining or comparing two signals. For this kind of application, it is more interesting to use a fixed regular partition with a known approximation ability.

From a signal processing standpoint, the fuzzy transform framework looks like a sampling/interpolation process, classically used to solve continuous problems by discrete computations or to perform computations that are equivalent to continuous processes [15,35]. Most results reported in the fuzzy transform literature are classical signal processing results. Thus, how does this fuzzy framework apply in the signal processing context? As base functions? As a complementary tool?

A very interesting answer to this question was proposed by Perfileva in [23] whereby new fuzzy transforms based on residuated lattice operations were constructed. These new transforms lead to interval-valued direct and inverse fuzzy