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Lattice-valued preuniform convergence spaces ☆

Jinming Fang

Department of Mathematics, Ocean University of China, 238 Song Ling Road, Qingdao 266100, PR China

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Abstract

Using the intrinsic lattice-valued inclusion orders of L-subsets and of stratified L-filters, the concept of a stratified L-preuniform convergence structure is proposed. This convergence structure is asymmetric and can be used to establish a framework of asymmetric lattice-valued space structures. The category of stratified L-preuniform convergence spaces is introduced and some categorical properties are presented. A reflective subcategory of stratified L-preuniform convergence spaces is found that is categorically isomorphic to strong stratified L-convergence spaces (originally called stratified L-ordered convergence spaces). Several subcategories of stratified L-preuniform convergence spaces are established and relations between the different subcategories are presented. We conclude that stratified L-preuniform convergence structures introduced here could play a role in the framework of lattice-valued asymmetric space structures in lattice-valued topology.

Keywords: Stratified L-preuniform convergence space; Stratified L-preconvergence space; Stratified L-convergence space; Stratified L-topology; Stratified L-preuniform limit space; Stratified L-quasiuniform limit space; Stratified L-quasiuniform space; Categorical properties

1. Introduction

In many-valued (or lattice-valued) settings, there are different approaches to the generalized notion of a uniform structure, such as Lowen's uniformities [19], Höhle's L-fuzzy uniformities [12], Hutton's uniformities [11] and various generalizations, such as those proposed by Shi [25] and Yue and coworkers [27,28], among others. Relationships among these structures are described elsewhere [10,16,29]. Note that the space structures of these approaches are symmetric in the sense that different lattice-valued uniformities satisfy the so-called symmetry axiom. To guarantee natural-function space structures, Jäger and Burton introduced stratified L-uniform convergence spaces [18]. The lattice context of these spaces was later generalized from complete Heyting algebras to the case of enriched lattices [3]. Furthermore, to obtain nice structural properties, such as extension and the closedness of the product of quotient mappings, we previously introduced the category of stratified L-semiuniform (L-ordered semiuniform) convergence spaces [6], which is Cartesian-closed and closed under the formation of products of quotient mappings. Moreover, the category of stratified L-semiuniform convergence spaces is extensional.

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E-mail address: jinming-fang@163.com.

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It is well known that the category of stratified L-topological spaces is not completely satisfactory from a structural point of view, in that there is no natural-function space structure for the sets $C(X, Y)$ of morphisms. Hence, Jäger developed a theory of convergence based on the notion of stratified L-filters in the case in which L is a complete Heyting algebra [17]. Yao generalized this type of lattice-valued convergence to the lattice context of complete residuated lattices [26]. Orpen and Jäger generalized the lattice context further by considering enriched cl-premonoids [22], which further serve as a common lattice basis to capture both definitions of a lattice-valued filter [20,21,26]. The resulting category contains the category of stratified L-topological spaces as a reflective subcategory and has the desired structure property of Cartesian-closedness when the underlying lattice is a complete Heyting algebra [17]. When considering the compatibility of lattice-valued convergence structures with the L-inclusion order of L-subsets, Fang [5,7] and Li and Jin [20,21] independently, proposed a concept of stratified L-ordered convergence space (or stratified L-convergence space [20]). Moreover, Fang showed that the category of stratified L-ordered convergence spaces is also Cartesian-closed when the underlying lattice is a complete Heyting algebra [5]. We also found that the category of stratified L-ordered convergence spaces contains strong L-topological spaces [30] as a reflective subcategory when the underlying lattice L is a commutative unital quantale [7]. Hereafter, a stratified L-ordered convergence structure and stratified L-ordered convergence space are called a strong stratified L-convergence structure and strong stratified L-convergence space, respectively.

In the classical case, by omitting the symmetry axiom, Preuss obtained the concept of a preuniform convergence space and then the category **PUConv** of preuniform convergence spaces [24], which offers a unified visual perspective for understanding both quasiuniform convergence structures (including quasiuniform structures) and generalized convergence structures, as well as topologies. In light of this, we introduce the concept of stratified L-preuniform convergence space and the category of stratified L-preuniform convergence spaces. Some nice categorical properties are presented. The characteristic of a stratified L-preuniform convergence structure is compatible with the L-inclusion order of stratified L-filters.

We know that the category **GConv** of generalized convergence spaces is categorically isomorphic to a bicoreflective subcategory of the category **PUConv** of preuniform convergence spaces [24]. In lattice-valued settings, it is interesting to know if the category of strong stratified L-convergence spaces could be embedded into that of stratified L-preuniform convergence spaces. To this end, according to the compatibility of space structures with the L-inclusion order of stratified L-filters, we introduce the concept of a stratified L-preuniform convergence space, and the category of stratified L-preuniform convergence spaces and some nice categorical properties are presented. By looking for a subcategory of the category of stratified L-preuniform convergence spaces, namely the category of stratified L-preconvergence spaces, we prove that the category of strong stratified L-convergence spaces is isomorphic to that of stratified L-preconvergence spaces, and the latter is a bicoreflective subcategory of the category of stratified L-preuniform convergence spaces. In this way, the category of strong stratified L-convergence spaces proposed by Fang [5] and Li and Jin [20] using different names and notions can be considered as a subcategory and studied in the framework of stratified L-preuniform convergence spaces.

We also study several subcategories of stratified L-preuniform convergence spaces when the underlying lattice is an enriched Heyting algebra and the relations that exist between the different types of subcategories. These results enable us to recognize that stratified L-preuniform convergence structures can offer a framework of asymmetric lattice-valued space structures in lattice-valued topology.

It should be pointed out that the L-inclusion order of L-subsets and of stratified L-filters are types of intrinsic structures in fuzzy set theory, and hence we should make good use of this theory in studying lattice-valued (or fuzzy) topology. The difference between the approach in previous studies on lattice-valued uniform convergence structures and the approach here is that instead of the classical pointwise order structure (inherited from the underlying lattice L) of stratified L-filters, the intrinsic L-inclusion order structure of stratified L-filters, which enables us to define compatible lattice-valued space structures, play a decisive role here.

The remainder of the paper is organized as follows. Section 2 describes the background in terms of notations and results for stratified L-filters. In Section 3, we propose the concept of stratified L-preuniform convergence spaces and the category of stratified L-preuniform convergence spaces, and present some nice categorical properties. Section 4 focuses on embedding the category of strong stratified L-convergence spaces into the category of stratified L-preuniform convergence spaces. Section 5 discusses subcategories of the category of stratified L-preuniform convergence spaces, and relations between the different types of subcategories are established. Section 6 contains concluding remarks and a diagram of relations between various subcategories of the category of stratified L-preuniform convergence spaces.