Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 266 (2015) 84–100

FUZZY sets and systems

www.elsevier.com/locate/fss

A novel algorithm for fusing preference orderings by rank-ordered agents

Fiorenzo Franceschini [*], Domenico Maisano, Luca Mastrogiacomo

Politecnico di Torino, DIGEP (Department of Management and Production Engineering), Corso Duca degli Abruzzi 24, 10129, Torino, Italy

Received 3 April 2014; received in revised form 2 August 2014; accepted 16 September 2014

Abstract

Yager [22] proposed an algorithm to combine multi-agent preference orderings of several alternatives into a single consensus fused ordering, when the agents' importance is expressed through a rank-ordering and not a set of weights. This algorithm is simple and automatable but has some limitations which reduce its range of application, e.g., (i) preference orderings should not include incomparabilities between alternative and/or omissions of some of them, and (ii) the fused ordering may sometimes not reflect the majority of the multi-agent preference orderings.

The aim of this article is to present an enhanced version of the Yager's algorithm, which overcomes the above limitations. Some practical examples support the description of the new algorithm.

Keywords: Decision making; Multi-agent; Preference ordering; Fusion; Ordinal semi-democratic; Partial ordering

---

1. Introduction

A general problem, which may concern practical contexts of different nature, is to aggregate multi-agent orderings of different alternatives into a single consensus fused ordering. Assume that there are M decision-making agents D_1, $D_2, \ldots, D_M$, each of which defines an ordering of n alternatives a, b, c, etc. For any two alternatives a and b, this ordering allows statements like $a > b$, $a \sim b$, $b > a$, where symbols "$>$" and "$\sim$" respectively mean "strictly preferred to" and "indifferent to".

This decision-making problem is very diffused [21,23] in a variety of real-life contexts, ranging from *multi-criteria decision aiding* [9,10] to *social choice* [14] and *voting theory* [3,6]. Two of the reasons for this diffusion are that (i) preference orderings are probably the most intuitive and effective way to represent preference judgments of alternatives, and (ii) they do not require a common scale – neither numeric, linguistic nor ordinal – to be shared by the interacting agents [5,22].

---

[*] Corresponding author.

E-mail addresses: fiorenzo.franceschini@polito.it (F. Franceschini), domenico.maisano@polito.it (D. Maisano), luca.mastrogiacomo@polito.it (L. Mastrogiacomo).

http://dx.doi.org/10.1016/j.fss.2014.09.011
0165-0114/

The literature embraces a variety of aggregation techniques, which are relatively interchangeable among the fields of application. Despite this variety, they can generally be divided into two categories [1]:

1. Methods in which all agents have the same importance [23]; e.g., let us consider the classical approaches in the *voting theory* field [2,7,15];
2. Methods in which agents have recognized abilities and attributes and/or privileged positions of power, represented by weights [8,21]; e.g., let us consider the ELECTRE or the PROMETHEE methods, in the *multicriteria decision aiding* field [4,10].

Considering the second category methods, the definition of the agents' weights is sometimes controversial, because there are no indisputable criteria for this operation. Although the literature includes several techniques about the quantification of weights – for example, the AHP procedure [20] or the method proposed by Martel and Ben Khelifa [17] – these techniques are often neglected because of their complexity.

In some situations weights are not available or cannot be defined. In this contexts agent's importance may be modeled by a simple rank-ordering, such as $D_1 > (D_2 \sim D_3) > \ldots > D_M$ [11,19]. The formulation of such a rank-ordering is certainly simpler and more intuitive than that of weights, especially when the agent's importance prioritization is doubtful [5]. This more specific decision-making framework can be denominated as "ordinal semi-democratic"; the adjective "semi-democratic" indicates that agents do not necessarily have the same importance, while "ordinal" indicates that their hierarchy is defined by a crude ordering. The set of the possible solutions to the problem may range between the two extremes of (i) *full dictatorship* – in which the fused ordering coincides with the preference ordering by the most important agent (dictator) – and (ii) *full democracy* – where all agents' orderings are considered as equi-important.

Despite the adaptability to a large number of practical contexts, this specific decision-making problem has received relatively little attention in the literature. Several years ago, Yager [22] proposed an algorithm to address this problem in a relatively simple, fast and automatable way; this algorithm will be hereafter abbreviated as YA, which stands for "Yager's Algorithm". Unfortunately, this algorithm has two major limitations: (i) the resulting fused ordering may sometimes not reflect the preference ordering for the majority of agents [13] and (ii) it is only applicable to (non-strict) *linear* orderings (such as the one exemplified in Fig. 1(a)) without incomparabilities and omissions of the alternatives of interest. These and other limitations will be clarified in the next section.

The objective of this paper is to enhance the YA so as to overcome its limitations and adapt to less stringent preference orderings (such as that one exemplified in Fig. 1(b)). The new algorithm will be denominated as "Enhanced (Yager's) Algorithm" – hereafter abbreviated as EYA – and can be interpreted as a generalization of that by Yager.

The remainder of the paper is organized into three sections. Section 2 recalls the YA in detail, with special attention to its limitations. Section 3 illustrates the EYA, highlighting its advantages with respect to the YA. The description of both algorithms is supported by practical examples. The concluding section summarizes the original contributions of the paper and its practical implications, limitations and suggestions for future research.

2. Yager's algorithm (YA)

In Section 2.1 we take the liberty to illustrate the YA from a "pedagogical" point of view. For a more rigorous description, we refer the reader to the original contribution by Yager [22]. Section 2.2 discusses the (dis)advantages of this algorithm.

2.1. YA description

The algorithm can be schematized in the following three basic phases, which are described individually in the Sections 2.1.1 to 2.1.3:

- construction and reorganization of preference vectors;
- definition of the reading sequence;
- construction of the fused ordering.