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Membership function construction for interval-valued fuzzy sets with application to Gaussian noise reduction

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Abstract

Models based on interval-valued fuzzy sets make it possible to manage numerical and spatial uncertainty in grey-scale values of image pixels. In a recent paper, we proposed a method that links the ultrafuzziness index (that makes it possible to take into account some uncertainty, like noise, and inherent to image capture) with impulse noise removal. However, computing with interval-valued fuzzy sets requires assigning their membership functions (MFs). The present article proposes a novel method for the generation of membership functions, based on image histogram, to remedy that drawback and it complements our previous study. The performance of the method is evaluated by applying this technique to the particular case of Gaussian noise detection and reduction, which remains a crucial issue for image processing. Experimental results have demonstrated that the proposed method leads to interval-valued fuzzy filters that are comparable with some well-known conventional and fuzzy filters, especially in the case of iterative filtering methods. Image details are preserved while reducing Gaussian noise, and the link between image noise and interval-valued fuzzy sets is thus verified. The main advantage of the proposed technique is to use basic image information, namely an image histogram, which is easy to obtain.

Keywords: Interval-valued fuzzy sets; Ultrafuzziness index; Kernel smoothing; Image filtering; Gaussian noise

1. Introduction

In this paper, we propose a new method to generate fuzzy membership functions for image pre-processing (it particularly addresses the classic image denoising problem). An interval-valued fuzzy set (IVFS in the sequel, first introduced by Zadeh [1]) is defined by an interval-valued membership function (we shall refer to the MFs associated with IVFSs as IVMFs).

In a previous paper [2] we have shown that (generalized) ultrafuzziness index (which is also called IVFS entropy in this paper) is a particularly adapted tool to detect relevant information in noisy images, and makes impulse noise

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removal from images possible. IVFSs enable to model the lack of knowledge of the expert when constructing the membership function of fuzzy sets (FSs) used for image denoising. We have also shown [3] that this method is efficient to remove speckle noise. We have recently applied it to define a no-reference quality metric of computer-generated images [4] (and its application to denoising). These interesting results raise the question: is it possible to automatically generate membership functions of IVFS by using image information? The issue of the automatic generation of IVMFs from images is an important step in developing algorithms to handle uncertainties. Thus in the present paper we develop the method we previously proposed and define a new generic frame to construct IVFSs (for image processing). We also investigate the optimization of the parameters involved in this algorithm, and we illustrate this new concept through Gaussian noise estimation and removal in digital images.

Fuzzy sets are interesting tools to evaluate a data element along with the information contained in its neighborhood [2,5–7] and they make it possible to manage the imprecision that is present in discrete images. Typical membership functions of fuzzy sets are essentially generated based on heuristics (experts, ...), probability, entropy and histograms. IVFSs were introduced to deal with other dimensions of uncertainty. One way to model uncertainty in the pixel values of a grey-scale image is by using fuzzy mathematical morphology (FMM) [8,9]. Extensions of FMM were then proposed. Extension of FMM include bipolar fuzzy sets (where positive and negative aspects of information are defined [10]), interval-valued FMM [11] and general L-fuzzy mathematical morphology [12]. Nachtegael et al. [11] successfully investigated the construction of IVFS for image processing using these tools. Sussner et al. presented an approach for edge detection based on an interval-valued morphological gradient [13,12], and obtained good results when applying this method to medical image processing.

Another way to model uncertainty in grey-scale images is to use fuzzy relations. Barrenechea et al. [14,15] presented a new construction method for interval-valued fuzzy relations and they applied it to image processing with success. Jurio et al. [16] proposed to use the local information provided by pixel neighborhood for the construction of IVFSs. Choi et al. [17] proposed to generate interval type-2 fuzzy MF for pattern recognition application, by using feature histograms of the input data.

Some authors have successfully developed noise detection methods in discrete images [18,19] based on fuzzy or neuro-fuzzy techniques. Similarly, we propose to use the IVFS filter defined in [2] which allows for the construction of IVFSs to obtain an efficient noise detector. We propose a novel **iterative algorithm** to generate IVMFs. This method is based on ultrafuzziness function thresholding and is relatively simple and computationally efficient. IVFS is constructed from the association of two non-additive Gaussian kernels permitting noise detection and estimation. Then the upper and lower bounds of the MF are obtained from two smoothed histograms.

Gaussian noise most often contaminates image during acquisition procedure, and it is commonly assumed that each pixel has to be corrected by removing the random additive error, which makes image restoration very difficult in this case.

Nachtegael et al. [5] have compared 38 different algorithms (conventional and fuzzy filters) on their performances w.r.t. Gaussian noise in grey-scale images. They proved that from a numerical point of view, the best performing filters are all based on fuzzy logic. Nevertheless, they showed that the visual results could still be improved for high level Gaussian noise. Thus, the application part of this paper considers the removal of this kind of high-level noise in grey-scale images, and illustrates the IVFSs construction method we propose with good results.

The paper is organized as follows: Section 2 briefly describes some preliminary definitions and IVFS filter, Section 3 introduces the IVMF generation method, and Section 4 presents some results. Conclusion and potential future work are considered in Section 5.

2. Preliminary definitions

The concept of an interval-valued FS was first introduced by Zadeh [1]. In standard fuzzy set theory, a fuzzy set F (over the space X) is defined by a membership function such $\mu(x) \in [0, 1]$ [20]. We recall in the case of an interval-valued fuzzy set A, the membership grade M_A is expressed as an interval rather than as a precise point (A is defined over a crisp universe of discourse X, [1,15]):

$$M_A(x) = [\mu_L(x), \mu_U(x)] \quad (1)$$