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Fuzzy quantification: a state of the art

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Abstract

Quantified sentences are a very powerful notion for modelling statements in Natural Language (NL), but in practice they have been used to solve several problems. This paper is intended to offer a global view of the development on this branch until now, focusing in the different approaches dealing with quantification, specially those involving imprecision, called fuzzy quantification. We put attention to the different mechanisms for defining them, the evaluation methods for measuring their fulfilment, as well as the properties they should satisfy.

Keywords: Fuzzy quantification; Quantified sentences; Protoform; Evaluation of quantified sentences; Natural language processing; Semi-fuzzy quantifier; Quantifier fuzzification mechanism

1. Introduction

Quantified statements are applied in the resolution of a great variety of problems not only in Natural Language (NL) modelling. Much of these problems used them for representing assertions and/or restrictions involving the quantity or the percentage of objects verifying a certain property. In addition, humans use NL statements in their reasoning processes. Because of this, many authors have tried to develop a model for the representation of this knowledge from several perspectives.

Historically, first works considering linguistic quantification are those of Mostowski [63] where the existential and universal quantifiers are studied from an operational point of view. Later, Rescher in a brief two pages note gives the seed for considering quantification apart from the *all* or *exists* quantifiers [70]. In the AI field first works were made by Zadeh [91,92,94] using of the theory of fuzzy sets. In these works Zadeh recognized the necessity of using fuzzy sets for representing more types of quantifiers than the existential (∃) and universal (∀) quantifiers used in First Order Logic which symbolize respectively that *at least one* object satisfies a property; and that *all* the objects satisfy the specified property. Quantifiers representing notions such as “a few”, “a lot” or proportions like “around a half” or “most” are intrinsically imprecise and fuzzy sets seem to be an adequate tool for modelling them.

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The notion of linguistic quantifier modelled by means of fuzzy sets was then proposed for modelling the so-called *fuzzy quantified sentences*. Quantifiers are usually grouped in two different types. Depending if they represent imprecise quantities or proportions they are classified into *absolute* or *relative* quantifiers respectively. Examples of absolute quantifiers are "around 2", "approximately between 1 and 3", "exists", representing imprecise quantities or intervals. Relative quantifiers represent fuzzy percentages as for instance "at least a half", "most" and "approximately a half". These quantifiers can be seen as fuzzy numbers defined as normalized, convex fuzzy sets defining restrictions on their respective domains. Depending on the type of quantifier the sentences are also classified into two classes: type I sentences ("Q of X are A") and type II sentences ("Q of D are A") where X is a finite crisp set, Q is a linguistic quantifier and A, D are fuzzy properties defined over X. An example of type I sentence is "Most of the students are young" and of type II is "Most of the good students are young" where X is a finite set of students, the quantifier is "most" the set A is the property "young" and the set D represents the property "good".

In parallel, although they were less known in the AI ambit, we can find works following a linguistic perspective using the Theory of Generalized Quantifiers [3,76,58], TGQ for short. The TGQ recognizes more than 30 types of quantifiers that can be unary ("Roger is A") or multiplace ("few As are Bs"), quantitative ("about ten") or non-quantitative ("all but John"); and they can be simple ("many") or complex or constructed ("most married As are Bs or Cs"). In this theory the quantifiers are also known as *determiners*.

Later, a generalization of the TGQ was proposed by Glöckner [36] using a procedure to transfer crisp quantifiers to their fuzzy analogous. The transformation procedure called Quantifier Fuzzification Mechanism (QFM) has several benefits: (1) it is not limited to only absolute or relative quantifiers, (2) it deals with multiplace quantifiers (e.g. more A than B) and (3) it extends the TGQ. In fact, we will see that the type I and II sentences can be seen as particular types of quantified sentences within this approach.

Note that some quantifiers are well defined from a crisp perspective, e.g. "Exists A" it can be defined as

$$\exists A = \begin{cases} 1 & A \neq \emptyset, \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

but others are intrinsically vague or fuzzy in nature, e.g. "few D are A" and the accomplishment degree of the sentences involving them cannot be modelled by a function defined over the $\{0, 1\}$ bi-valued set. In these cases it is more appropriate to use fuzzy sets for representing them and computing the accomplishment degree yielding a value in the unit interval $[0, 1]$. In addition, fuzzy quantifiers are often easily well understood for humans (as well as their graphical representation).

The assessment of quantified sentences is a matter of degree, and hence the semantic concepts that we can define using them are also fuzzy concepts. This is natural since quantifiers and the sets D and A are fuzzy sets. The process of calculating the accomplishment degree of a quantified sentence is usually known as an evaluation method. This paper is then intended to collect and review all the evaluation methods in fuzzy quantification developed until now. These approaches are divided in two different sections to distinguish those following the notation of type I and II sentences and those following the extension proposed by Glöckner in the TGQ. But although in the literature we can find such two frameworks, the Theory of Generalized Quantifiers includes and extends the so-called type I and II sentences that are related to absolute and relative quantifiers.

To facilitate the reader in choosing the best evaluation method that better fits to his necessities, we offer not only the methods but also the properties that should be verified by them. The reader can find here also a complete discussion that helps to understand the relations between the methods and under what assumptions are they defined. Several papers present a set of desirable properties for an evaluation method [20,4,2,22,38,41]. We analyze all these properties as well as the equivalences between the different proposals of properties and the methods satisfying them.

One of the main reasons of this compilation is due to the huge amount of areas of applicability of linguistic quantified sentences. In information retrieval quantified sentences are used to model natural language sentences for querying a database in a more flexible manner. In data mining they offer a suitable tool for measuring the assessment of different kinds of association rules. Different evaluation methods have been used with satisfactory results in linguistic summarization, linguistic description of data and many others, which are presented in a separate section in order to have a wide vision of their vast applicability in several domains. We go more deep in a recent line of research very related to the sentence evaluation which is that of reasoning with quantified sentences. Notice that this new dimension of reasoning taking into account the imprecision is being also developed in other fields such as Description Logics where the fuzzy evaluation methods are being used in the process. We conclude the paper with a brief discussion and stressing the main lines for future work.