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A logic framework for reasoning with movement based on fuzzy qualitative representation [☆]

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Abstract

We present a logic approach to reason with moving objects under fuzzy qualitative representation. This way, we can deal both with qualitative and quantitative information, and consequently, to obtain more accurate results. The proposed logic system is introduced as an extension of Propositional Dynamic Logic: this choice, on the one hand, simplifies the theoretical study concerning soundness, completeness and decidability; on the other hand, provides the possibility of constructing complex relations from simpler ones and the use of a language very close to programming languages.

Keywords: Qualitative reasoning with fuzzy data; Moving objects; Order-of-magnitude reasoning; Propositional dynamic logic; Reasoning under uncertainty

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1. Introduction

Qualitative reasoning is an interesting tool in order to deal with incomplete information which sometimes happens when we are dealing with moving objects. Some papers have been published which study and develop qualitative kinematics models [8,19,21,37], following the ideas presented in [11,12,26]. Different approaches have been used in order to face the problem of the relative movement of one physical object with respect to another [9,10,36]. However, to the best of our knowledge, the only paper which introduces a logic framework to manage qualitative velocity is [5].

Sometimes, using just qualitative reasoning is not precise enough, especially when we have to take into account precise absolute locations that may be known in advance in some applications [22]. In particular, this can be a problem in specific tasks related to moving objects, such as collision avoidance, catching an object, etc. As a consequence, a combination of qualitative and quantitative data would be required, and it seems that Fuzzy Qualitative Reasoning (FQR) can be a good choice for that purpose [34]. FQR uses fuzzy numbers in order to represent qualitative classes and can be applied to robot kinematics [19] by using fuzzy qualitative trigonometry [20]. Several recently published

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papers develop different applications of FQR to human motion [6], dynamic systems [7], geographical systems [16], Fuzzy Spatial Reasoning [30–32].

On the other hand, fuzzy logic controllers have been designed and used to improve navigation in mobile robots, presenting a set of IF-THEN rules to formulate the attributes of human reasoning and decision-making [27,28]. Furthermore, a fuzzy control system for reactive navigation of mobile robots has been presented recently in [25].

In this paper, we continue the line of [5] by presenting a logic approach to deal with moving objects with fuzzy qualitative representation. We exploit the advantages of using fuzzy numbers in qualitative reasoning for simplifying the tables of compositions of movements. In some sense, this choice allows for using both qualitative and quantitative information, and consequently, obtain more accurate results. The use of logic improves also the capability of formal representation of problems and provides insights into their most suitable solving methods. As examples of logics for qualitative reasoning see, for example [23,29]. Our logic approach is based on Propositional Dynamic Logic (PDL) because it provides the possibility of constructing complex relations from simpler ones and the use of a language very close to programming languages. We will exploit these advantages of PDL by giving specific programs for collision avoidance. We choose PDL which is a decidable logic and, as a consequence, we have the advantage that reasoning can be performed by theorem proving. Some applications of PDL in AI can be seen in [3,4,35].

The present approach focuses on the movement of objects with respect to others with or without obstacles. Our aim is to develop a formalism capable of indicating any relative position of an object with respect to another one, in such a way that allows us to calculate different movements and represent certain actions in the chosen scenarios. We are specially interested in representing specific actions such as collision avoidance and intercepting an object. In order to establish a sufficiently detailed and accurate calculus, this formalism is integrated into the logic PDL. This choice is highly pertinent because PDL is an excellent tool for managing operations on these vectors and represent the dynamism of actions.

We represent the movement of an object with respect to another by a tuple whose components include information about objects, velocity, orientation, relative movement, allowed movements, qualitative latitude and longitude. Some of these components were inspired by previous works in the literature whereas others have been included in order to increase the expressive power of our approach. For instance, [10] uses two components, for velocity and orientation, which are considered as relative magnitudes; our approach considers velocity and orientation as absolute magnitudes, because so are the values obtained from devices such as velocimeters, GPS, etc. On the other hand, [9] considers two components as well, but their interpretation is different: the relative movement and the relative velocity of one object with respect to another; the former is included in our approach.

The advantages of our approach are two-fold: on the one hand, it subsumes in some sense several previous approaches, and the formalization provided by the logic allows reasoning without using too many case-based tables; on the other hand, our approach is flexible enough so that the number and/or the specifications of the components of a movement can be modified without altering much the general framework: for instance, the components of relative position and cardinal direction could be enriched in the line of [24,33].

The paper is organized as follows: Section 2 introduces the preliminary definitions and notations to be used in the rest of the paper; Section 3 is devoted to present our approach in different scenarios used in the literature; in Section 4, we introduce our logic approach to reasoning with moving objects with fuzzy qualitative data; then, in Section 5, we present our logic approach and provide a set of specific programs for collision avoidance; the soundness, completeness and decidability of the proposed logic are studied in Section 6; finally, we draw some conclusions and prospects of future work.

2. Preliminary definitions

We represent the movement of an object with respect to another with different labels such as velocity, orientation, relative movement, possible directions and relative position. The values of these labels are given by different qualitative classes, and the granularity can be changed depending on the problem in question. To be more precise, we represent the movement of an object $\mathsf{A_i}$ with respect to $\mathsf{A_j}$ by $(x_1; \ldots ; x_7) \in L$, being $L = L_1 \times \cdots \times L_7$ defined as follows. As some of the sets L_i are defined also by a Cartesian product, for an easy reading we will eliminate some parentheses by using “;” to indicate the seven components of our label, while we will use “,” for the components of each L_i. In this case, we will use superscripts to each element of this Cartesian product. For example $x_7 = (x_7^1, x_7^2)$.