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Fuzzy Sets and Systems 297 (2016) 46–72

FUZZY sets and systems

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Fuzzy rule base simplification using multidimensional scaling and constrained optimization

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Received 6 February 2014; received in revised form 14 October 2015; accepted 27 October 2015

Available online 3 November 2015

Abstract

This paper proposes a novel approach for the development of highly accurate and interpretable fuzzy models with Gaussian fuzzy sets, under the framework of multidimensional scaling and non-linear constrained optimization. Upon the assumption that an accurate initial fuzzy model has already been designed, we introduce an effective methodology to approximate the similarity measure between fuzzy sets. The resulting similarity degrees guide the quantification of the dissimilarity degrees between rule antecedents. We, then, put the multidimensional scaling in place in order to transform the rule antecedents into points in a low dimensional Euclidean space. The elaboration on the distribution of these points is carried out by means of an objective-function based fuzzy clustering using a cluster validity index. Rules that correspond to points belonging to the same cluster are similar and therefore, are unified through a specialized merging process. With respect to each dimension, the aforementioned merging process acts to create a topology of fuzzy sets that enables us to elicit interpretability constraints, which are used to minimize the model's performance index in terms of non-linear constrained optimization. The established fuzzy model appears to possess a simple and transparent (i.e. interpretable) structure while maintaining a highly accurate behavior. The overall method is rigorously tested and evaluated through a number of simulation experiments that involve low and high dimensional data sets.

Keywords: Fuzzy modeling; Interpretability; Accuracy; Multidimensional scaling; Approximated similarity measures; Non-linear constrained optimization

1. Introduction

During the last decades, fuzzy modeling has been extensively exploited as an alternative to more traditional modeling considerations [4,5,15,18,19,45,49]. The reason that gave impetus to this tendency relies on two basic structural characteristics the fuzzy models possess. First, they are universal approximators and second, their inherent configuration supports the processing of linguistic concepts [8,14,21,42]. The former concerns the accurate model's behavior,

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http://dx.doi.org/10.1016/j.fss.2015.10.009

Nomenclature

A_{ij}	Fuzzy set of the ith rule in the jth dimension
a_{ij}	Consequent parameters of the initial fuzzy model
b_{ij}	Consequent parameters of the simplified fuzzy model
C_i	Fuzzy clusters in the low dimensional transformed Euclidean space
c	Number of rules of the simplified model
$c_{\max}$	Maximum number of clusters used to apply the cluster validity index
d_{il}	Euclidean distance in the low dimensional transformed Euclidean space
E	Area
f_{kj}	Distinguishability constraint functions
G	Functions that aggregate the constraints used in the constrained optimization
g_{kj}	Completeness constraint functions
h_i	Fuzzy basis functions
J_{SC}	Multidimensional scaling objective function
J_{SE}	Square error objective function
M	Number of rules of the initial fuzzy model
N	Number of training data
p	Number of dimensions of the feature space
Q	Number of equidistant points that discretize each universe of discourse
R_i	The ith fuzzy rule
R_i^A	The antecedent part of the ith fuzzy rule
$\boldsymbol{r}_i$	Vectors that represent the cluster centers in the low dimensional transformed Euclidean space
S	Similarity measure
s_{ij}	Fuzzy set widths of the simplified fuzzy model
UoD_j^L	Lower limiting point of the jth universe of discourse
UoD_j^U	Upper limiting point of the jth universe of discourse
u_{ik}^{SC}	Membership degrees of the clusters in the transformed Euclidean space
v_A	Center of the fuzzy set A
v_{ij}	Fuzzy set centers of the initial fuzzy model
$\boldsymbol{v}_i^{SC}$	Cluster centers in the low dimensional transformed Euclidean space
w_i	Firing strength of the ith fuzzy rule
$\boldsymbol{x}_k$	The kth input training data vector
x_{AB}	Intersection point between the fuzzy sets A and B
z_i	The consequent part of the ith fuzzy rule
y_k	The kth output data
$\tilde{y}_k$	Fuzzy model's estimated output
β_{ij}	Constraints involved in the optimization procedure
γ	Penalty factor
δ_{il}	Dissimilarity measure between rule antecedents
ϑ_A^L	The lower point that corresponds to a small membership degree ξ of the fuzzy set A
ϑ_A^U	The upper point that corresponds to a small membership degree ξ of the fuzzy set A
μ_{ij}	Type of membership function involved in the cluster validity index
ξ	A small number corresponding to the ξ-cut of a fuzzy set
σ_A	Width of the fuzzy set A
σ_{ij}	Fuzzy set widths of the initial fuzzy model
υ_{ij}	Fuzzy set centers of the simplified fuzzy model
ϕ_{ij}	Constraints involved in the optimization procedure
φ_{ij}	Constraints involved in the optimization procedure
$\boldsymbol{\chi}_i$	Data vectors resulting from the multidimensional scaling transformation
$\bar{\boldsymbol{\chi}}$	Mean value of all $\boldsymbol{\chi}_i$
ψ_{kj}	Equidistant points that discretize each universe of discourse
Ω_j	Set of the fuzzy set widths in the jth dimension

while the latter reflects on model's ability to generate and reform human-understandable knowledge and is usually referred to as interpretability [13,24,27,29,37,47]. It has been widely acknowledged that although these two characteristics are of major importance, their direct implementation appears to obtain conflicting results. The experience acquired so far has shown that this contradictory behavior is by far the most challenging problem in fuzzy modeling. Yet, this problem indicates the very core of the diversification of fuzzy modeling from other modeling strategies, since it directly designates the model's potential to provide user friendly knowledge; an issue that can hardly be addressed by other modeling schemes. Finding a balance between interpretability and accuracy appears to be a tough problem mainly because of the varying model's structure/behavior from application to application [7,10,16,22,24,51,52]. Al-