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The structure of generalized intermediate syllogisms ☆

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Abstract

In this paper, we continue development of the formal theory of intermediate quantifiers which are expressions of the natural language ("most","many","few", etc.). In the previous paper, we demonstrated that 105 generalized syllogisms are valid in our theory. We introduce the generalization of all the figures and we show that for the proof of validity of all the generalized syllogisms, we need to prove the validity of only few of them so that the validity of the other ones immediately follows.

Keywords: Fuzzy type theory; Generalized quantifiers; Intermediate quantifiers; Logical syllogisms

1. Introduction

The concept of "fuzzy quantifiers" has been developed by Zadeh in [22]. This theory has been later elaborated by Glöckner in [5]. The intermediate quantifiers have been also studied in [18]. Intermediate quantifiers are expressions of natural language such as *most, many, almost all, a few, a large part of*, etc. Semantically they characterize number (or quantity) of elements having a certain property over a universe. In fact, they refine the quantification to lay between the limit cases *for all* (∀) and *exists* (∃) — hence the name. This class of quantifiers was deeply studied by Peterson in the book [17] from the point of view of their semantics and general logical properties. A reasonable formalization of intermediate quantifiers was first given in [11]. Its basic idea consists in the assumption that intermediate quantifiers are just classical quantifiers ∀ or ∃ but the universe of quantification is modified. In classical logic there can hardly be found any substantiation for such modification. The clue is provided by the theory of evaluative linguistic expressions (see [10]) developed as a special theory of the higher-order fuzzy logic. Recall that the former are expressions such as "very small", "roughly big", "more or less medium", etc. Using them, we can characterize size of the universe over which the quantification proceeds. Consequently, because the meaning of them is imprecise, the meaning of the intermediate quantifiers is imprecise as well. Moreover, this imprecision, which is in accordance with our intuition, is naturally obtained.

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A closer inspection of the formalism of intermediate quantifiers reveals that they are construed using special formulas consisting of two parts:

(i) Characterization of the size of a given fuzzy set using specific measure and some evaluative linguistic expression.
(ii) Ordinary quantification (general or existential) of the resulting formula.

The formulas are constructed in a certain extension of the special formal theory T^{Ev} of FTT which describes semantics of *trichotomous evaluative linguistic expressions*.

The formal theory of intermediate quantifiers has several outcomes. First of all, we do not need to define semantics of intermediate quantifiers by introducing new special symbols with specific interpretation. Furthermore, the structure of the large class of quantifiers is characterized in a unique way. But even more, let us recall that one of the results of [17] is demonstration of validity of 105 syllogisms that generalize the classical Aristotle ones. Thus, we can take it as a benchmark so that every other theory of intermediate quantifiers must be able to verify validity of these syllogism as well. We focused on this problem in [8] where we proved that all 105 syllogisms from [17] are valid also in our formal theory and that syllogisms assumed to be invalid in the former are invalid also in the latter. Moreover, the validity is strong in the following sense: let P_1 and P_2 be premises and C a conclusion. Then the truth of the conjunction $P_1 \,\&\, P_2$ is smaller than or equal to the truth of the conclusion C in every model.

It must be emphasized that our formal theory is very strong because it is formulated syntactically. Hence, because of the completeness theorem, our results are valid *in all* models. Therefore, our theory encompasses also a great deal of results obtained semantically in the literature on fuzzy and generalized quantifiers.

The reader who carefully read the paper [8] must have noticed that there are certain regularities in the proofs of validity of the syllogisms. Moreover, we considered in that paper only several specific quantifiers and considered syllogisms containing them. But the formal theory is general enough to include a much larger class of quantifiers and this raises a question under which conditions we can assure that any new syllogism will also be valid. This is the topic of this paper.

In Section 2 we briefly overview fuzzy type theory and the theory of evaluative linguistic expressions. Section 3 is an overview of the basic concepts and properties of the theory of intermediate quantifiers. Section 4 contains an analysis of the structure of generalized Aristotle's syllogisms. We generalize all the syllogisms for arbitrary kinds of intermediate quantifiers. We demonstrate that in each figure, there is one basic syllogism the validity of which implies the validity of several other ones.

2. Preliminaries

The formal system within which we develop our theory is higher-order fuzzy logic called, in accordance with the classical logic, *fuzzy type theory*. Among several kinds of the latter that depend on the chosen structure of truth values, we us the Łukasiewicz fuzzy type theory Ł-FTT. In this section, we will briefly overview the main concepts and properties of Ł-FTT and the formal theory of evaluative linguistic expressions.

2.1. Syntax of Ł-FTT

Types and formulas

The basic syntactical objects of Ł-FTT are classical — see [1], namely the concepts of *type* and *formula*. The atomic types are ϵ (elements) and o (truth values). General types are denoted by Greek letters $\alpha, \beta, \ldots$. Complex types are formed iteratively as sequences $\beta\alpha$. The set of all types is denoted by *Types*. The *language* of Ł-FTT, denoted by J, consists of variables $x_\alpha, \ldots$, special constants $c_\alpha, \ldots$ ($\alpha \in Types$), the symbol λ, and brackets. It is specific for Ł-FTT that connectives are also formulas. Special formulas (connectives) introduced in Ł-FTT are *fuzzy equality/equivalence* $\Leftrightarrow$, *conjunction* $\wedge$, *strong conjunction* $\&$, *disjunction* $\vee$, *implication* $\Rightarrow$ and the delta $\boldsymbol{\Delta}$.

Ł-FTT has 17 axioms and two deduction rules which can be found in [8]. Recall that the rule of modus ponens and the rule of generalization are derived rules. The concepts of *provability* and *proof* are defined in the same way as in classical logic. A *theory* T over Ł-FTT is a set of formulas of type o ($T \subset Form_o$). By $J(T)$ we denote the language of the theory T. By $T \vdash A_o$ we mean that A_o is provable in T.

The following lemma will be used later.