Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 247 (2014) 127–134

FUZZY sets and systems

www.elsevier.com/locate/fss

Fuzzy t-filters and their properties

Martin Víta [a,b,*,1]

[a] *Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod Vodárenskou věží 2, 182 07 Prague, Czech Republic*
[b] *NLP Centre, Faculty of Informatics, Masaryk University, Botanická 68a, 602 00 Brno, Czech Republic*

Received 4 January 2013; received in revised form 22 September 2013; accepted 3 November 2013

Available online 8 November 2013

Abstract

Fuzzification of special types of filters on several different algebras of many-valued logics has been very popular in recent years. The main aim of this paper is to point out some general principles concerning particular results about fuzzification of special types of filters. We introduce the notion of a *fuzzy t-filter* which generalizes most types of special fuzzy filters (e.g. fuzzy implicative, fuzzy boolean, fuzzy fantastic, etc.) and prove some basic properties of fuzzy t-filters.

Keywords: Non-classical logics; Algebra; Fuzzy t-filters; t-filters; Fuzzy filters; Residuated lattices

1. Introduction and preliminaries

Filters play an important role when studying algebraic semantics of different logics. The notion of a filter was fuzzified in a natural way and then a huge amount of papers about special types of fuzzy filters was published in many journals.

The motivation of this paper is primarily to show that fuzzification of special types of filters can be done uniformly by our simple theory and there is no need to present 'new' analogous particular results for those special types of filters which have not been fuzzified yet. Our goal is not to produce another paper about special types of fuzzy filters, but cover such particular papers with this unifying approach.

As a side effect, these results provide a tool for reviewers dealing with papers about particular new special types of fuzzy filters.

Basic settings: Residuated lattices and filters

The definition of a t-filter was introduced in [1] in order to generalize some results about special types of filters mentioned in the crisp case. We are going to recall this definition for residuated lattices.

* Correspondence to: Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod Vodárenskou věží 2, 182 07 Prague, Czech Republic. Tel.: +420 604 619 669; fax: +420 286 585 789.

E-mail address: info@martinvita.eu.

1 The support of RVO 67985807 and the grant P202/10/1826 of the Czech Science Foundation is acknowledged. The author wishes to express thanks to his former supervisor Petr Cintula for valuable comments and helpful suggestions.

0165-0114/$ – see front matter
http://dx.doi.org/10.1016/j.fss.2013.11.002

At the very beginning we recall definitions of a residuated lattice and a filter.

Definition 1. A *bounded pointed commutative integral residuated lattice* is a structure

$$\boldsymbol{L} = (L, \&, \to, \wedge, \vee, \bar{0}, \bar{1})$$

of type (2, 2, 2, 2, 0, 0) which satisfies the following conditions:

(i) $(L, \wedge, \vee, \bar{0}, \bar{1})$ is a bounded lattice.
(ii) $(L, \&, \bar{1})$ is a monoid.
(iii) $(\&, \to)$ form an adjoint pair, i.e. $x\&z \leqslant y$ if and only if $z \leqslant x \to y$ for all $x, y, z \in L$.

According to the common practice we usually omit the words 'bounded, pointed commutative integral' when talking about these structures and use simply the notion 'residuated lattices'. The terminology in this field is a bit confusing: for precious distinctions of similar structures see [2]. Our approach corresponds with a custom practice in the field of fuzzy logic topics (see [3] for instance).

Starting now we assume that $\boldsymbol{L}$ is a residuated lattice and L is its domain.

Definition 2. A non-empty subset F of L is called a *filter* on $\boldsymbol{L}$ if following conditions hold for all $x, y \in L$:

(i) if $x, y \in F$, then $x\&y \in F$,
(ii) if $x \in F$, $x \leqslant y$, then $y \in F$.

The definition of a filter on L can be given by many equivalent ways, for comprehensive overview see [3] again.

Basic notions of fuzzy set theory and fuzzy filters

For any $\alpha \in [0, 1]$ and an arbitrary fuzzy set α we denote the set $\{x \in X \mid \mu(x) \geqslant \alpha\}$ (i.e. the α-cut) by the symbol μ_α.

Definition 3. A fuzzy set μ of L is a *fuzzy filter* on $\boldsymbol{L}$ if and only if it satisfies the following two conditions for all $x, y \in L$:

(i) $\mu(x\&y) \geqslant \min\{\mu(x), \mu(y)\}$,
(ii) if $x \leqslant y$, then $\mu(x) \leqslant \mu(y)$.

The definition of a fuzzy filter can be again formulated by many equivalent ways, see [3]. Sometimes in the further text we use the assumption that a fuzzy filter μ satisfies $\mu(\bar{1}) = 1$ which means that the fuzzy set μ is *normal*, i.e. its height is equal to 1. It would be useful to call these fuzzy sets *normal fuzzy filters*, but due to terminological reasons (possible confusion with fuzzy normal filters) we will not use this terminology.

The relationship between fuzzy filters and (crisp) filter on the same algebra $\boldsymbol{L}$ can be described in terms of α-cuts.

Theorem 4. *(See [3].) A fuzzy set μ on $\boldsymbol{L}$ is a fuzzy filter if and only if for each $\alpha \in [0, 1]$ the (crisp) set μ_α is either empty or a filter on $\boldsymbol{L}$.*

Definition of a t-filter

This subsection presents results from [1] which are used in the further text and provides some examples t-filters.

For the reader's convenience and better clarity of presented proofs we use the following convention: by the symbol $\bar{x}$ we denote the abbreviation of $x, y, \ldots$, i.e. $\bar{x}$ is a formal listing of variables used in a given context. Thanks to this convention we can properly use the notation $\bar{x} \in L$ instead of $x, y, \ldots \in L$.

By the term t it is always meant a term in the language of residuated lattices. In this context we also recall common convention: $\neg x$ is an abbreviation of $x \to \bar{0}$.