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Tunable equivalence fuzzy associative memories

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Abstract

This paper introduces a new class of fuzzy associative memories (FAMs) called *tunable equivalence fuzzy associative memories*, for short tunable E-FAMs or TE-FAMs, that are determined by the application of parametrized equivalence measures in the hidden nodes. Tunable E-FAMs belong to the class of Θ-FAMs that have recently appeared in the literature. In contrast to previous Θ-FAM models, tunable E-FAMs allow for the extraction of a fundamental memory set from the training data by means of an algorithm that depends on the evaluation of equivalence measures. Furthermore, we are able to optimize not only the weights corresponding to the contributions of the hidden nodes but also the contributions of the attributes of the data by tuning the parametrized equivalence measures used in a TE-FAM model. The computational effort involved in training tunable TE-FAMs is very low compared to the one of the previous Θ-FAM training algorithm.

Keywords: Parametrized equivalence measure; Fuzzy associative memory; Tunable equivalence fuzzy associative memory; Selection of fundamental memories; Supervised learning; Classification

1. Introduction

The class of Θ-fuzzy associative memories (Θ-FAMs) represents a class of fuzzy associative memories (FAMs) with a competitive hidden layer and with weights that can be tuned using a specific training algorithm [1].

A Θ-FAM is determined by functions Θ^ξ that are applied in the ξth hidden node. Previous publications on Θ-FAMs have focused on the cases where the functions Θ^ξ are given by fuzzy subsethood or similarity measures, leading to (weighted) subsethood, dual subsethood, and similarity measure FAMs [1,2]. These models, referred to using the acronyms S-FAMs, dual S-FAMs, and SM-FAMs (or weighted S-FAMs, dual S-FAMS, and SM-FAMs if one wishes to stress the inclusion of adjustable weights in their first layers), have found inspiration in fuzzy mathematical morphology (FMM) [3–6]. In this context, recall that in FMM the elementary operator of fuzzy erosion is defined in terms of the degree of inclusion or subsethood of objects (i.e., translated versions of the so called structuring element) in another object (given by the image under consideration) [7]. In addition, certain similarity measures employed

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in (weighted) SM-FAMs can be related to fuzzy hit-or-miss transforms. Therefore, (dual) subsethood and similarity measure FAMs can be viewed as fuzzy morphological neural networks, more specifically as *fuzzy morphological associative memories* (FMAMs) [8].

Previously, many well-known fuzzy associative memories (FAMs) from the literature had already been classified as FMAMs [8] since they perform elementary morphological operations in the complete lattice setting [9–11] of fuzzy mathematical morphology [6]. Examples of FMAMs include Kosko's max–min and max-product FAMs [12], the generalized FAM of Chung and Lee [13], Junbo's FAM [14], the max–min FAM with threshold of Liu [15], the fuzzy logical bidirectional associative memory of Belohlávek [16], as well as implicative fuzzy associative memories [17]. All of these FAM models represent fully-connected fuzzy neural networks without hidden layers [18,19]. As mentioned before, Θ-FAMs are equipped with a competitive hidden layer. In this respect, Θ-FAMs are similar to the well-known Hamming net [20] or Hamming associative memory and its extensions [21,22].

The aforementioned Θ-FAM models, i.e., weighted S-FAMs, dual S-FAMs, and SM-FAMs, were successfully applied to a number of classification problems and to a problem of vision-based self-localization in robotics [1]. The tuning of the weights was performed using a training algorithm that is guaranteed to converge in a finite number of steps and, under some weak conditions, to reach a local minimum of the proposed objective function. On the downside, the present Θ-FAM training algorithm is computationally very expensive, turning its use infeasible on very large datasets. Another option for tuning the weights of the aforementioned Θ-FAM models would be the use of a derivative-free non-linear optimization method such as a genetic algorithm. However, the application of a genetic algorithm to problems involving a large number of variables often becomes impracticable due to its high computational cost [23].

Here, we propose a computationally efficient alternative for tuning the weights of certain Θ-FAM models that are introduced in this paper. The first stage of the methodology proposed in this paper consists in extracting relevant fundamental memories from the training data by means of a selection algorithm that can only be applied to these types of Θ-FAMs. More precisely, this algorithm is based on the evaluation of parametrized equivalence measures, that can be chosen to serve as functions Θ^ξ in a Θ-FAM model. The resulting Θ-FAM models are named *tunable equivalence fuzzy associative memories*, for short, *tunable E-FAMs*. If the underlying equivalence measures are differentiable with respect to their parameters, then the parameters and the weights of a tunable E-FAM can be tuned in the second stage of our methodology using a conventional nonlinear optimization algorithm [24].

Since equivalence measures can be defined on any bounded lattice, we begin by providing some background information on lattice theory and equivalence measures. Section 3 introduces tunable E-FAM models in the context of Θ-FAMs. Section 4 describes our strategy for clustering the training data in terms of equivalence measures and for training the weights and parameters of tunable E-FAM models. Section 5 is concerned with the application of tunable E-FAMs to several classification problems from the KEEL and UCI databases [25,26]. Section 5 reveals that the classification rates produced by the tunable E-FAM algorithm compared favorably to the ones obtained by other classifiers for these problems in the recent literature [27,28]. Finally, we finish this paper with some concluding remarks in Section 6.

2. Mathematical background

In 1965, Lotfi A. Zadeh extended the classical notion of a set, also referred to as a crisp set, by introducing fuzzy sets [29]. A fuzzy set A consists of a set X, called universe, together with a membership function $\mu_A : X \to [0, 1]$ that yields the membership degree $\mu_A(x) \in [0, 1]$ for each $x \in X$. The symbol $\mathcal{F}(X)$ denotes the class of fuzzy sets on the universe X.

Mathematically speaking, a fuzzy set A on a universe X can be identified with its membership function μ_A and can simply be viewed as a function from the universe X to the unit interval $[0, 1]$. If X is finite, say $X = \{x_1, \dots, x_n\}$, then $\mathcal{F}(X)$ can be identified with $[0, 1]^n$ via the bijection that maps $A \in \mathcal{F}(X)$ to the vector $(A(x_1), \dots, A(x_n))^t \in [0, 1]^n$. Note that the notion of a crisp set arises if A (or, more precisely, μ_A) only adopts values in $\{0, 1\}$. In particular, the classical concept of a relation is given by a subset of $X \times Y$, where X and Y are arbitrary universes, and can thus be viewed as a function $X \times Y \to \{0, 1\}$. Extending this classical concept to the fuzzy domain, a fuzzy relation is given by a function $R : X \times Y \to [0, 1]$, where $R(x, y)$ can be interpreted as the degree of relationship between x and y [30]. In other words, a fuzzy relation on $X \times Y$ is nothing else than a fuzzy set on $X \times Y$.