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Interpretability of fuzzy linguistic summaries

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This work is dedicated to Francesc Esteva, for his pioneering work in fuzzy logic and his dedication to the fuzzy community, especially in Europe

Abstract

This paper investigates the question of the interpretability of fuzzy linguistic summaries, both at the sentence level and at the summary level, seen as a set of sentences. The individual sentence interpretability is examined as depending both on its representativity measured by a quality degree and on its linguistic expression. Different properties at the summary level are also discussed, namely their consistency, their non-redundancy and the information they convey.

Keywords: Fuzzy linguistic summaries; Interpretability; Summary consistency; Generalised protoform

1. Introduction

The interpretability of information handled automatically by machines has always been a difficult problem. For instance, the attempts of humans to extract knowledge from large amounts of data or to interpret the functioning of complex systems motivated researches in domains such as association rules in databases or expert systems, to name a few. The increasing size of available data in the digital world and the diversity of medias and kinds of data, be they images, time series or databases, for instance, put pressure on researchers to provide efficient methods to mine and summarise information in an easily understandable form, to help users such as decision makers or medical doctors to manage efficiently complex cases.

Fuzzy set-based methods are certainly useful in such perspectives for their capacity to process linguistic information through the interface they provide between numerical and symbolic values, and also for their intrinsic ability to reduce complexity by providing a synthesis of individual elements. They enable users to have more friendly interactions with machines than many other methods of computational intelligence. Their interpretability has notwithstanding been strongly questioned and methods to improve it have been proposed.

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Interpretability of fuzzy models is a very complex criterion, difficult to define precisely, partly subjective and depending on the context of utilisation. It has been extensively studied in the case of fuzzy rule-based systems [1–4], which have been the most widespread within all fuzzy modelling methods, mainly because of the early success of fuzzy expert systems and fuzzy control, both based on rules of the form "if V_1 is A_1 and V_2 is A_2, ... , then W is B". These studies point out the difficulty to assess interpretability and provide various kinds of properties involved in its evaluation. Compactness, completeness, consistency on the one hand, coverage, normality and distinguishability of fuzzy modalities on the other hand, are considered as the most relevant properties interesting to maximise when looking for a trade-off between expressibility of fuzzy rule-based systems and accuracy of the model.

A growing interest has been expressed more recently for fuzzy linguistic summaries that provide a textual description of numerical data. They have been introduced decades ago and are more and more studied because of the nowadays difficulty to grasp efficiently all available digital information. Textual representation of information can be more efficient than graphical ones in several cases. For instance, the data can be described in high dimensional domains, possibly and generally speaking hard to show graphically, in which case the linguistic summary is an interesting alternative [5]. Moreover, it has been shown that information displayed as text to the user is interpreted more swiftly compared to graphs [6]. Finally, a linguistic summary can be read out by a text-to-speech synthesis system when the visual attention must not be disturbed, while executing a complex task for instance [7], or when it is deficient.

As remarked in [8], the interpretability of fuzzy linguistic summaries has not been much studied as compared to fuzzy rule-based systems, probably for two reasons. First of all, less attention has been brought to them than fuzzy rule-based though they have been introduced in 1982 [9] and mainly investigated since 2001 [10]. Secondly, there exist various kinds of linguistic summaries going beyond the protoforms originally proposed and capturing the generality of the concept is not an easy task.

We can distinguish two points of view to analyse the interpretability of linguistic summaries. The first one considers a process of linguistic summarisation taking into account a list of generalised protoforms based on a number of quantifiers or temporal indications appropriate for the addressed problem, and fuzzy characterisations based on partitions of universes of definition of attributes. In this framework, the above-mentioned properties involved in the interpretability of fuzzy rule-based models are still useful to evaluate with regard to both lists of quantifiers or temporal indications and fuzzy characterisations.

Another point of view, especially relevant in the case of general and more complex forms of fuzzy summaries, takes into account more various elements used to construct them and their interpretability then lies on external considerations corresponding to their acceptability by users or their relevance in terms of natural knowledge.

In this paper, after discussing the general definition of fuzzy linguistic summaries in Section 2, we propose to investigate the question of the interpretability of fuzzy linguistic summaries following a two level approach: first, at the sentence level in Section 3, then at the summary level, seen as a set of sentences in Section 4.

2. Fuzzy linguistic summaries

Fuzzy linguistic summaries can be defined as texts made of several sentences that describe distinct characteristics of a given data set.

The individual sentences that make up the summary are most of the time seen as schemata that are instantiated with respect to the considered data set, a schema being called a *protoform* [9,11]. Many variants have been proposed, from "Qy's are A" and "QBy's are A" [9] to more complex ones, as discussed below. Their variety both depends on the type of information they extract from the data and on the type of data they apply to, e.g. data described by numerical attributes or times series.

We propose to consider *generalised protoforms* as "MBy's are A" where M is a *mode*, that describes the extent to which "By's are A" holds for the considered data, adjusting its meaning. M can be either a quantifier, as in classic protoforms [9,11], or a temporal indication in the case where the data are time series, as discussed below. A and B are fuzzy modalities, respectively called *summariser* and *qualifier*, of linguistic variables used to describe the data; y denotes elements of the dataset to be summarised.

Classic protoforms The notion of protoforms [11] using quantifiers [12] applied to summaries [9] has been introduced in the case of numerical attribute data, as "Qy's are A" or more generally "QBy's are A". They can be