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Closeness in similarity-based reasoning with an interpolation condition

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Abstract

This study considers an approximate reasoning scheme where the knowledge base is a set of fuzzy IF–THEN rules and the inference mechanism is characterized in terms of closeness. We propose the interpretation of this scheme, which uses Ruspini's theory of conditional consistency and implication measures, and its adaptation to fuzzy sets. In the proposed interpretation, we find the necessary and sufficient conditions to ensure that the computation of the conclusion fulfills the interpolation condition. We show that the inference mechanism is equivalent to the compositional rule of inference in the form of inf → composition. Finally, we construct a particular interpretation where all of the considered requirements are satisfied and the inference operator reduces to a piecewise linear interpolation function.

Keywords: Approximate reasoning; Closeness; Compositional rule of inference; Interpolation; Similarity

1. Introduction

A general approximate reasoning scheme is expressed in the following form

$$\begin{array}{llll} \textit{given a set of IF–THEN rules:} & \text{IF}\ \ X \text{ is } A_i & \text{THEN}\ \ Y \text{ is } B_i, & i=1,\dots,n, \\ \textit{and a fact} & \quad\ X \text{ is } A, & & \\ \textit{infer a conclusion} & & \quad\ \ Y \text{ is } B, & \end{array} \tag{1}$$

based on the *meta-inference rule*, [6,9,3]

$$\text{the } \textit{closer} \text{ the input } A \text{ is to } A_i, \text{ then the } \textit{closer} \text{ the output } B \text{ is to } B_i. \tag{2}$$

If $A, B, A_i, B_i, i = 1, \dots, n$, are fuzzy predicates, then the scheme (1) + (2) is usually known as the *generalized modus ponens* [4,23]. When supplied with a proper interpretation, this scheme provides an estimation of a truth qualification of the conclusion B.

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The main aim of the present study is to construct an interpretation of (1) + (2) such that the estimated conclusion B coincides with B_i whenever the fact A coincides with corresponding A_i, $i = 1, \dots, n$. We say that this interpretation is *interpolative*. Under interpolative interpretation, the scheme (1) + (2) coincides with the modus ponens for $A_1, \dots, A_n$ as antecedents. This fact explains why the interpolation property has been added as a constraint.

We note that the generalized modus ponens is the most widely used approximate reasoning scheme because it has numerous applications in fuzzy control and decision making. In [10], many other approximate reasoning schemes are discussed in connection with the notion of similarity. The latter is used in the present study for the interpretation of closeness.

The interpretation of (1) + (2) can be constructed in many ways, depending on the selection of the set of truth values, logical operations, aggregation of rules, and interpretation of closeness. When it has been constructed, it determines what can be considered as an *inference operator*, i.e., an operator that assigns a conclusion B to each fact denoted by A. However, for every $i = 1, \dots, n$, the inference operator does not always return B_i if the fact A coincides with A_i, i.e., the constructed interpretation is interpolative. Therefore, the interpretation should be restricted in an appropriate manner in order to guarantee this property.

In this study, we search for the interpretation of (1) + (2) such that the set of truth values and logical operations are taken from a residuated lattice L, where the facts $A, A_1, \dots, A_n$ and conclusions $B_1, \dots, B_n$ are L-valued fuzzy sets on X and Y, respectively,[1] and in the case where $A = A_i$, the corresponding inference operator returns B_i. We refer to this problem as *interpolative approximate reasoning* (**IAR**).

The **IAR** problem has been investigated intensively in previous studies of approximate reasoning, fuzzy sets, and fuzzy logic in a broader sense (e.g., see [13,20,3,2,5,10,17,21,23]). From a theoretical perspective (important for the theory of approximate reasoning), a solution to **IAR** determines two spaces with corresponding closeness relations (they characterize a quality of approximation) and an inference operator that establishes a type of morphism (due to preservation of closeness), which passes through the given samples. From an applied perspective, the formal expression of a solution to **IAR** represents a generic case of an interpolation surface, which can be reduced to a particular interpolation function after specifying the residuated lattice and corresponding similarities.

Next, we provide a brief overview of some major contributions to the study of **IAR**. This is presented in two sections because the **IAR** combines two interpretations: interpretations of a set of IF–THEN rules and of an inference mechanism. If both are selected independently, then the interpolation condition is not guaranteed, which requires that additional requirements should be imposed.

IF–THEN rules In [18], the two parts of an IF–THEN rule are combined by the min operation and the whole set is aggregated by the max operation. This interpretation together with the max–min composition for deriving a conclusion is still very successful in applications. This approach is supported by system engineers but it has been criticized by logicians (see [7]).

Another (straightforward) approach (e.g., see [19,4]) combines the IF and THEN parts of the IF–THEN rule using an implication operation and min is employed for aggregation. As noted in [7], the first interpretation is natural for representing positive information (adding observations), while the second provides an appropriate reflection of the combination of negative information (rules, constraints, etc.).

Rule of inference According to the original approach described in [23], an inference rule has the form of a (max–min) composition between a fuzzy set and a fuzzy relation that interprets the set of IF–THEN rules. This form is called a *compositional rule of inference*, which is abbreviated to (max–min) CRI. Subsequently, other CRIs (e.g., sup−$*$, inf $\rightarrow$) were proposed and analyzed (e.g., see [1,12,8]). Based on the CRI, the inferred conclusion is a result of a pure computation that involves fuzzy sets, relations, and operations over them. The CRI is used in applications where positively represented information is processed. It is not explicitly connected with any linguistic characterization. If the CRI is chosen, then the property formulated in (2) requires verification.

The second approach to the interpretation of the rule of inference (2) is based on a pure (literal) translation of its linguistic characterization. This translation requires the interpretation of two measures of closeness and their comparison. Based on Ruspini's theory of conditional consistency and implication measures [22], two (infinite) parametric

[1] We assume that the reader is familiar with the basic notions of the L-fuzzy set theory.