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Fuzzy Sets and Systems 239 (2014) 3–20

FUZZY sets and systems

www.elsevier.com/locate/fss

Do inferential processes affect uncertainty frameworks?

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Available online 5 July 2013

Abstract

This paper studies the connections among different (comparative or numerical) degrees of belief. In particular we consider, in turn, a comparative probability or possibility on a given Boolean algebra and we prove that their upper extensions to a different Boolean algebra are, respectively, a comparative plausibility or possibility. On the other hand, in general the upper extension of a comparative necessity is simply a comparative capacity. Moreover, by considering a suitable condition of weak logical independence between the two Boolean algebras, we prove that the upper ordinal relation is a comparative possibility in all the aforementioned cases. We consider specifically also the lower ordinal relations, since they may not be the comparative dual relation of the upper ones.

Keywords: Ordinal relations; Extendibility; Probability; Possibility theory

1. Introduction

If a decision maker or a field expert has only information concerning events different from those of interest, it is necessary to get information on the latter, i.e. to make inference. More precisely, given an assessment consisting of a degree of belief referring to a specific framework (singled-out by a suitable uncertainty measure), making inference means extending the given assessment to new events. Since in general the extension is not unique, to avoid introducing arbitrary information or intermediate arbitrary choices of the decision maker during the inferential process, we must consider all the extensions and then their upper and lower envelopes.

For syntactical reasons the lower and upper envelopes of the enlargements can lead to uncertainty measures or binary relations which are different from the initial ones. In other words, we could move from a framework to another one.

We deal with the inferential problems starting from different (numerical or comparative) degrees of belief on a given Boolean algebra and look for their upper and lower extensions to another Boolean algebra (possibly related to the first one).

As far as numerical assessments are concerned, we recall for instance the following result: if we start from a probability P on a Boolean algebra $\mathcal{A}$, and consider any other Boolean algebra $\mathcal{B}$, then the lower [upper] envelope of the class of probabilities extending P to $\mathcal{B}$ is a belief [plausibility] function (see e.g. [20,31,9]). As proved in [10],

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0165-0114/$ - see front matter
http://dx.doi.org/10.1016/j.fss.2013.06.014

under suitable logical conditions between the two Boolean algebras (weak logical independence), the upper envelope is a possibility and the lower envelope is a necessity.

Notice that this notion of weak logical independence is meaningful also from a practical point of view, see Examples 1 and 2 (and also [11]).

We deepen some numerical aspects (dealt with in [10]) by showing that the upper envelope of possibilities is still a possibility and the lower envelope of necessities is still a necessity (thus, these measures maintain the same properties of the initial measure). On the contrary, in general both the lower envelope of possibilities and the upper envelope of necessities are simply a capacity (so only monotonicity is preserved), while under weak logical independence the lower envelope reduces to a necessity and the upper envelope reduces to a possibility. Furthermore we prove that a suitable pair of capacity and possibility can be seen as the lower and upper envelopes of a possibility. Similar results are obtained when we start from a necessity.

The main interest of this paper, which is an extension of [12], is directed toward inferential problems in a comparative setting. In fact, the decision maker may have at hand (or is able to give) not a numerical assessment, but only a binary relation expressing the idea of "not more believable than" (comparative degree of belief). The aim is to extend the comparative degree of belief from a family of events to another one (i.e. to make inference).

We focus on the extension of specific comparative degree of beliefs: comparative probability [16], comparative possibility, comparative necessity [21]. The lower and upper envelopes of a comparative probability are comparative belief and plausibility, respectively (for definitions of comparative beliefs and plausibility, see [39]): this result is not trivial, since in general a comparative probability is not representable by a probability [33]. Analogous results hold even if we consider binary relations satisfying conditions weaker than those of qualitative additivity (the axiom characterizing comparative probabilities).

Lower and upper extensions of a comparative possibility are studied: the lower extension is simply a comparative capacity, while the upper extension is a comparative possibility.

On the other hand, under weak logical independence, the upper extension is always a comparative possibility and the lower envelope is a comparative necessity, independent of the specific property of the given comparative degree of belief. This emphasizes the fundamental role of weak logical independence in inferential processes.

These results contribute to the deepening of hybrid models involving various frameworks, such as probability, plausibility and possibility. Hybrid models are studied in many papers, e.g. [18,23,24,27–29,36]: our approach focuses on the inferential point of view in a comparative setting.

In Section 2 we recall some results on specific ordinal relations and on weak logical independence. In Section 3, after some preliminaries on coherent probability, we deal with possibility and its extensions, in particular we study the properties of lower and upper envelopes of extensions: we show that the lower envelope of the extensions of a possibility is simply a capacity. Moreover, any pair of capacity and possibility satisfying a specific condition can be seen as the lower and upper envelopes of the extensions of a possibility. In Section 4 the main results concerning the comparative setting are given: a characterization of lower and upper envelopes of extensions of some comparative degrees of belief is studied both under weak logical independence or not: we consider the extensions of comparative probability, necessity and possibility. In particular, we show that the lower envelope of extensions of a comparative probability is not a comparative probability, but a comparative belief.

2. Preliminaries

In this section we start (Section 2.1) by recalling some definitions and results related to binary relations among events, their extension and their representability by a numerical uncertainty measure. Then, in Section 2.2, we recall a condition (introduced in [10,11]) "weaker" than logical independence.

2.1. Comparative degree of belief

An event E is singled-out by a (nonambiguous) proposition, that is a statement that can be either true or false. Since in general it is not known whether E is true or not, we are uncertain on E. Two particular events are the certain event Ω (the top element) and the impossible event $\emptyset$ (the bottom element).

Notice that, directing attention to events just as subsets of the sample space (and to algebras of subsets) may be unsuitable for many real situations. It is instead very significant to give events a more general meaning. Even if any