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On fuzzy generalized convex mappings and optimality conditions for fuzzy weakly univex mappings ☆

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Abstract

In this paper, we first introduce the weakly invex fuzzy mappings based on weakly differentiable functions and discuss the relationships between several kinds of fuzzy generalized convex mappings. Then a kind of fuzzy weakly univex functions are introduced and some properties of them are investigated. Furthermore, optimality and duality results are derived for a class of generalized convex optimization problems with fuzzy weakly univex functions.

Keywords: Fuzzy optimization; Fuzzy weakly univex mapping; Sufficiency; Duality

1. Introduction

Convexity plays a vital role in many aspects of mathematical programming including, for example, sufficient optimality conditions and duality theorems. The fundamental property of a differentiable convex function $f : R^n \to R$ is its characterization given by the inequality

$$f(x) - f(y) \geq \nabla f(y)^t (x - y) \tag{1}$$

for all $x, y \in R^n$, where ∇ denotes the gradient. In a more general case, a function (not necessarily differentiable) is said to be convex on a convex set $X \subseteq R^n$ if for any $x, y \in X$, $\lambda \in [0, 1]$,

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda) f(y). \tag{2}$$

Moreover, (1) and (2) are equivalent for differentiable functions.

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In inequality constrained optimization, the Kuhn–Tucker conditions are sufficient for optimality if the functions involved are convex. However, application of the Kuhn–Tucker conditions as sufficient conditions for optimality is not restricted to convex problems, and various generalizations of convexity have been made in order to explore the extent of this applicability.

An invex function is one of the generalized convex functions and this was introduced by Hanson [15]. He considered a differentiable function $f : R^n \to R$ for which there exists a vector-valued function $\eta : R^n \times R^n \to R^n$ such that, for all $x, y \in R^n$, the inequality

$$f(x) - f(y) \geq \nabla f(y)^t \eta(x, y) \tag{3}$$

holds. Hanson [15] proved that if, instead of the usual convexity conditions, the objective function and each of the constraints of a nonlinear constrained optimization problem are all invex for the same η, then both the sufficiency of Kuhn–Tucker conditions and weak and strong Wolfe duality still hold. Later, Craven [11] named functions satisfying (3) invex (with respect to η).

Ben Israel and Mond [17] considered the preinvex function f with respect to η (not necessarily differentiable) for which there exists a vector-valued function $\eta : R^n \times R^n \to R^n$ such that, for all $x, y \in R^n$, the inequality

$$f(y + \lambda\eta(x, y)) \leq \lambda f(x) + (1 - \lambda) f(y) \tag{4}$$

holds. Moreover, they found that differentiable functions satisfying (4) satisfy (3), too. Further properties and applications of preinvexity and its some generalizations for some more general problems were studied by Antczak [2,3], Bector et al. [6], Mohan and Neogy [21], Suneja et al. [29], and others.

Later, a further generalization called type I function was considered by Hanson and Mond, and necessary and sufficient conditions for optimality of the primal and dual problems were given (Ref. [16]). The differentiable functions $f(x)$ and $g(x)$ are type I objective and constraint functions, respectively, with respect to η at a fixed point y if there exists an n-dimensional vector function η such that

$$f(x) - f(y) \geq \eta^t(x, y)\nabla f(y), \tag{5}$$

$$-g(y) \geq \eta^t(x, y)\nabla g(y). \tag{6}$$

Bector et al. [5] introduced the concept of univex functions as a generalization of B-vex functions introduced by Bector and Singh [4]. Let X be a nonempty open set in R^n, and let $f : X \to R$, $\eta : X \times X \to R^n$, $\Phi : R \to R$, and $b : X \times X \to R^+$, $b = b(x, y)$. A differentiable function f is said to be univex at $y \in X$ with respect to η, Φ, b if, for all $x \in X$

$$b(x, y)\Phi[f(x) - f(y)] \geq \eta^t(x, y)\nabla f(y). \tag{7}$$

Bector et al. [5] also introduced the concept of preunivex functions. A function f is said to be preunivex at $y \in X$ with respect to η, Φ, b if, for all $x \in X$

$$f(y + \lambda\eta(x, y)) \leq f(y) + \lambda b(x, y)\Phi[f(x) - f(y)], \tag{8}$$

where $\lambda b(x, y) \leq 1$.

Combining the concepts of type I and univex functions, Rueda et al. [27] gave optimality conditions and duality results for several mathematical programming problems. Aghezzaf and Hachimi [1] introduced classes of generalized type I functions for a differentiable multiobjective programming problem and derived some Mond–Weir type duality results under the above generalized type I assumptions. T.R. Gulati et al. [18] introduced the concept of $(F, \alpha, \rho, d) - V$-type I functions. They also studied sufficiency optimality conditions and duality for multiobjective programming problems.

On one hand, the fuzzy generalized convex mappings have been studied by many authors. For instance, in 1992, Nanda and Kar [23] proposed a concept of convex fuzzy mapping and proved that a fuzzy mapping is convex if and only if its epigraph is a convex set. In 1998, Furukawa [13] introduced convexity and local Lipschitz continuity concepts into the class of fuzzy-valued mappings. In their monograph, [26], Ramik and Vlach gave several types of generalized convex sets and generalized concave functions based on the support set of a fuzzy set. Yan and Xu [38] discussed convexity and quasiconvexity of fuzzy mappings by considering the concept of ordering proposed by